

Find the shear force and bending moments in the beam having the loading shown.

First, find $w_{AB}(x)$: $w_{AB}(x) = w_{\max}(1 - x/L_{AB})$

Finding the shear force for section AB:

$$V_{AB}(x) - V_A = - \int_{x'=0}^x w_{\max} \left(1 - \frac{x'}{L_{AB}}\right) dx'$$

$$V_{AB}(x) = -w_{\max} \left(x' - \frac{x'^2}{2L_{AB}}\right) \Big|_{x'=0}^{x'=x} = V_{AB}(x) = -w_{\max} \left(x - \frac{x^2}{2L_{AB}}\right)$$

Evaluate this at point B: $V_B = -w_{\max} \left(L_{AB} - \frac{L_{AB}^2}{2L_{AB}}\right) = \frac{-w_{\max} L_{AB}}{2} \Rightarrow V_B = -45N$

Finding the shear force for section BC. In this region, $w = 0$:

$$V_{BC}(x) - V_B = - \int_{x'=L_{AB}}^x 0 dx' = 0, \text{ so } V_{BC}(x) = \frac{-w_{\max} L_{AB}}{2}$$

Finding the bending moment for section AB:

$$M_{AB}(x) - M_A = \int_{x'=0}^x V_{AB} dx' = \int_{x'=0}^x -w_{\max} \left(x' - \frac{x'^2}{2L_{AB}}\right) dx' = -w_{\max} \left(\frac{x'^2}{2} - \frac{x'^3}{6L_{AB}}\right) \Big|_{x'=0}^x \Rightarrow M_{AB}(x) = -w_{\max} \left(\frac{x^2}{2} - \frac{x^3}{6L_{AB}}\right)$$

Evaluate this at point B: $M_B = -w_{\max} \left(\frac{L_{AB}^2}{2} - \frac{L_{AB}^3}{6L_{AB}}\right) = \frac{-w_{\max} L_{AB}^2}{3} \Rightarrow M_B = -90Nm$

Finding the bending moment for section BC:

$$M_{BC}(x) - M_B = \int_{x'=L_{AB}}^x V_{BC} dx' = \int_{x'=L_{AB}}^x -w_{\max} \frac{L_{AB}}{2} dx' = -\frac{w_{\max} L_{AB} x'}{2} \Big|_{x'=L_{AB}}^x = -\frac{w_{\max} L_{AB} x}{2} - \frac{-w_{\max} L_{AB}^2}{2} = -\frac{w_{\max} L_{AB} (x - L_{AB})}{2}$$

$$M_{BC}(x) = \frac{-w_{\max} L_{AB}^2}{3} - \frac{w_{\max} L_{AB} (x - L_{AB})}{2} = M_{BC}(x) = \frac{w_{\max} L_{AB}}{6} (L_{AB} - 3x)$$

The shear and bending moment diagrams are shown:

