Math Methods, Final Exam Equation Sheet — Dr. Pogo

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## Series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2!} + \frac{h^3 f'''(x)}{3!} + \dots$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S = \frac{a}{1-r}$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots$$

## Coordinate Systems

$$x = r\cos(q)$$
$$y = r\sin(q)$$
$$dA = rdrdq$$
$$ds = \sqrt{1 + r^2\left(\frac{dq}{dr}\right)^2}\,dr$$

$$dV = rdrdqdz$$
$$dA = rdqdz$$
$$ds^2 = dr^2 + r^2 dq^2 + dz^2$$

$$x = r\sin(q)\cos(f)$$
$$y = r\sin(q)\sin(f)$$
$$z = r\cos(q)$$
$$dV = r^2\sin(q)drdqdf$$
$$dA = r^2\sin(q)dq\,df$$
$$ds^2 = dr^2 + r^2dq^2 + r^2\sin^2(q)df^2$$

## ODE's

$$y' + Py = Q \rightarrow$$
$$I = \int Pdx$$
$$y = e^{-I}\int Qe^{I}dx + ce^{-I}$$

Given $r_1, r_2$:

$$y = Ae^{r_1x} + Be^{r_2x} \quad (r_1 \neq r_2)$$
$$y = (Ax+B)e^{r_1x} \quad (r_1 = r_2)$$
$$y = Ae^{r_1x} + Be^{r_2x} \quad (r_1, r_2 = a \pm ib)$$
or $y = e^{ax}\left(A\sin(bx) + B\cos(bx)\right)$

or $y = Ce^{ax}\sin(bx + g)$

$$y_p = \begin{cases} Ce^{cx} & \text{if } c \neq a,\ c \neq b \\ Cxe^{cx} & \text{if } c = (a \text{ or } b),\ a \neq b \\ Cx^2e^{cx} & \text{if } c = a = b \end{cases}$$

## Probability

$$p(AB) = p(A)p_A(B)$$
$$p(A+B) = p(A) + p(B) - p(AB)$$
$$P(n,r) = \frac{n!}{(n-r)!}$$
$$C(n,r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$
$$s^2 = \sum_i (x_i - m)^2 f(x_i)$$
$$= \int_{-\infty}^{\infty} \sum_i (x-m)^2 f(x)dx$$

Binomial: $f(x) = C(n,x)p^x q^{n-x}$

Normal: $f(x) = \frac{1}{s\sqrt{2p}} e^{\frac{-(x-m)^2}{2s^2}}$

$$s^2 \approx \frac{1}{n-1}\sum_{i=1}^{n}(x_i - m)^2$$
$$s_m = \frac{s}{\sqrt{n}}$$

Poisson: $P_n = \frac{m^n}{n!}e^{-m}$

## Inertia Tensors

$$I_{xx} = \sum_i m_i\left(y_i^2 + z_i^2\right) = \int\left(y^2 + z^2\right)dm$$
$$I_{xy} = -\sum m_i x_i y_i = -\int xydm$$

## Fourier

$$f(x) = \frac{1}{2}a_0 + a_1\cos(x) + a_2\cos(2x) + \dots$$
$$+ b_1\sin(x) + b_2\sin(2x) + \dots$$
$$a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos\left(\frac{np\,x}{\ell}\right)dx$$
$$b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin\left(\frac{np\,x}{\ell}\right)dx$$
$$f(x) = \int_{-\infty}^{\infty} g(a)e^{iax}da$$
$$g(a) = \frac{1}{2p}\int_{-\infty}^{\infty} f(x)e^{-iax}dx$$

## Derivatives & Integrals

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \dots$$
$$ds = \sqrt{dx^2 + dy^2}$$
$$ds = \sqrt{1 + \left(\frac{dy}{dy}\right)^2}\,dx$$
$$\overline{x} = \frac{\int xdm}{\int dm}$$
$$I = \int r^2 dm$$

## Complex Numbers

$$z = x + iy = re^{iq}$$
$$z = r\cos(q) + ri\sin(q)$$
$$\sin(q) = \frac{e^{iq} - e^{-iq}}{2i}$$
$$\cos(q) = \frac{e^{iq} + e^{-iq}}{2}$$
$$\sinh(z) = \frac{e^z - e^{-z}}{2}$$
$$\cosh(z) = \frac{e^z + e^{-z}}{2}$$

## Linear Algebra

Active Rotation Matrix: $\begin{bmatrix} \cos q & -\sin q & 0 \\ \sin q & \cos q & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Passive Rotation Matrix: $\begin{bmatrix} \cos q & \sin q & 0 \\ -\sin q & \cos q & 0 \\ 0 & 0 & 1 \end{bmatrix}$

## Vectors

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$
$$\nabla^2 f = \vec{\nabla}\cdot\vec{\nabla}f$$

$$\vec{V} = -\vec{\nabla}f$$
$$y = \int v_x dy = -\int v_y dx$$