ProofSpace Problem Set

Preliminaries

Logical Identities and Equivalences

Discussed Problems

1 Let a be a real number. What are the converse and contrapositive of the following statements?

a) If $a = 3$, then $a^2 = 9$.

b) If the phone rings, then I turn off the T.V.

c) If $a \neq 4$ or $a \neq -4$, then $a^2 \neq 16$.

2 Let P, Q, and R be statements. Use a truth table to prove the following equivalence:

$$P \wedge (Q \vee R) \quad \equiv \quad (P \wedge Q) \vee (P \wedge R).$$

3 Let P, Q, and R be statements. Use prior knowledge to prove the following logical equivalences.

a) $P \iff Q \quad \equiv \quad (\neg P \vee Q) \wedge (\neg Q \vee P)$.

b) $[P \wedge Q] \Rightarrow R \quad \equiv \quad (P \Rightarrow R) \vee (Q \Rightarrow R)$.

Evaluated Problems

1 Let P, Q, and R be statements. Use a truth table to prove the following equivalence:

$$P \vee (Q \wedge R) \quad \equiv \quad (P \vee Q) \wedge (P \vee R).$$

2 Let P, Q, and R be statements. Use prior knowledge to prove the following logical equivalences.

a) $\neg P \Rightarrow (Q \wedge \neg Q) \quad \equiv \quad P$.

b) $(P \Rightarrow Q) \Rightarrow R \quad \equiv \quad (P \wedge \neg Q) \vee R$

c) $\neg(P \vee Q) \Rightarrow R \quad \equiv \quad \neg(Q \vee R) \Rightarrow P$

Supplemental Problems

Mathematical Reasoning: Writing and Proof, Online Version 2.0, by Ted Sundstrom:
Sec. 2.2: 1, 2, 3, 6, 7, 8, 9, 10, 11

Advanced Problems

1 Use prior knowledge to prove that the following statement, called *reductio ad absurdum*, is a tautology:

$$((\neg P \Rightarrow Q) \wedge (\neg P \Rightarrow \neg Q)) \Rightarrow P.$$

2 (The Crocodile Dilemma) Suppose an extraordinarily indecent crocodile has stolen your write-up to this problem set (thereby actively hindering your ability to become the great prover you were meant to be). The crocodile promises you that he will return your write-up if and only if you can correctly predict whether or not the crocodile will return your write-up.

a) Find a prediction that leads to contradiction.

b) Show the crocodile who's boss. Show that the other prediction does not lead to contradiction.