# Some Pivotal Thoughts on the Current Balance

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The current balance is an excellent device for demonstrating the force on a current-carrying wire. By considering the electrons flowing through the wires and applying some geometrical analysis, we can gain a better understanding of why the wire moves, how the current is distributed in the wires, and why the simplifying assumptions of the force law apply to this realistic situation.

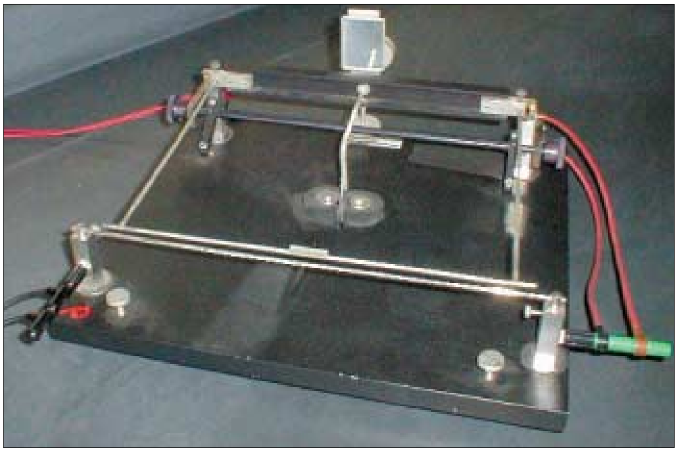

**Fig. 1. The current balance. With identical current flowing in opposite directions through the top, moveable wire and the bottom, stationary wire, the top wire will rise slightly.**

The current balance (Fig. 1) is a piece of equipment traditionally employed in an introductory electricity and magnetism laboratory course to illustrate the effects of Ampere's law. It consists of two parallel wires — a stationary wire mounted on an insulating base and a moveable wire that, through means of a counterweight and knife-edge fulcrum, is balanced just above the stationary wire.[1] When a current passes through each wire in opposite directions, the induced magnetic field produces a force on the moving charges and the wires repel. Typically, weights are added to the moveable wire until the system returns to the equilibrium position, so that the added weight is equal to the magnetic force on the wire. As such, the "Force on a Current-Carrying Wire" experiment is an excellent demonstration of the laws of electromagnetism. Indeed the SI unit of current (the ampere) and the unit of charge (the coulomb) are both defined based on the experiment involving two straight parallel conductors.[2]

This fundamental experiment has several subtleties, however, which uninitiated students may not discover. First of all, in textbooks and lectures, students are sometimes instructed that the magnetic field cannot do work. How is it, then, that the magnetic force upward results in an upward motion of the wire? In addition, in deriving the formula for the force between two current-carrying wires, the wires are assumed to be infinitesimally thin, when in fact the wires have some cross-sectional area. How is the current distributed across the cross-sectional face of the wires? How is the force acting on the moving electrons transferred to the wire itself? Finally, for a standard current balance, the wire diameter is about the same size as the separation between the wires. What effect does this have on the experiment?

 DOI: 10.1119/1.1571263 

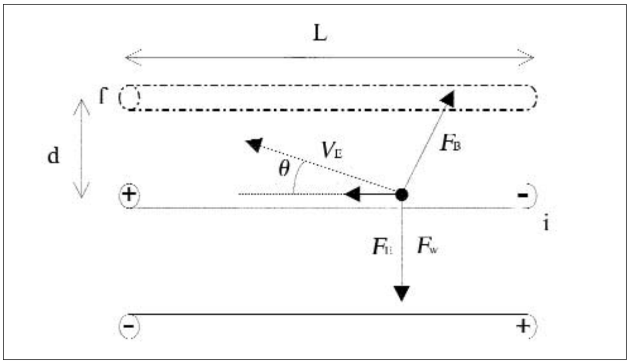


**Fig. 2. The forces acting on an electron in the moveable wire as that wire rises a distance *d*.**

## Magnetic Fields Can Do No Work

If a force acts on an object moving through some distance, the work done depends on the component of the force that is parallel to the direction of the displacement. The force provided by a magnetic field cannot do work because the force is always perpendicular to the displacement. Yet in this experiment there is an object (the moveable wire) that moves vertically in the region of the horizontal magnetic field. Where does the energy to lift the wire come from?

E.P. Mosca[3] has answered this question in the context of induced emf in a rod moving in an external magnetic field. The following argument is very similar to the one given in Ref. 3.

Figure 2 illustrates the case when the top wire is moving upward. Consider a particular electron in the moveable wire. Because the electron cannot escape the wire, it exerts a force on the wire ($F$). Consequently, there is an equal and opposite force acting on the electron due to the wire ($F_W$). As discussed in the appendix of Ref. 3, this force is due to the Hall effect. The force on the electron due to the electric field producing the current is $F_E$. The velocity of the electron has a horizontal component along the wire and a vertical component caused by the motion of the wire itself. The resulting magnetic force on the electron, $F_B$, is perpendicular to both the electron velocity and the magnetic field. These forces acting on the electron are shown in Fig. 2. Because the forces on the electron balance, we obtain, in the vertical direction,

$$F_W = F_B \cos\theta, \tag{1}$$

and in the horizontal direction,

$$F_E = F_B \sin\theta. \tag{2}$$

Eliminating $F_B$ from the two equations, we have

$$F_W = \frac{F_E}{\tan\theta} = \frac{eE}{\tan\theta}. \tag{3}$$

In the time it takes for the electron to travel a horizontal distance $L$, the wire moves a distance $d$, so that $\tan\theta = \frac{d}{L}$. Because $F_W$, the force of the wire acting on the electron is equal in magnitude to the force of the electron acting on the wire ($F$), the work done on the *wire* is

$$W_{wire} = F\,d = \left(\frac{eE}{\tan\theta}\right)d = \left(\frac{eE}{d/L}\right)d = eEL. \tag{4}$$

However, the work done by the power supply in moving an electron a distance $L$ along the wire is given by $eEL = eV$, where $V$ is the potential difference across that length of the wire. The work done by the power supply is equal to the work required to raise the wire. (See Ref. 3 for a brief discussion of a more complete picture.)

## Where Is the Current?

Since the moving electrons experience the magnetic force, wouldn't we expect the currents to be concentrated on the far sides of the wires? It seems reasonable that the currents repel one another in such a fashion.

Ohm's law is completely stated in terms of the current density $J$ as $J = \sigma(E + v \times B)$, where $v$ is the average velocity of the charge carriers in the wire. The $v \times B$ term leads to the Hall effect within the wire. However, this term is negligible when compared with the contribution from $E$. The potential difference between the two ends of the wire is given by $\Delta V = El$, where $l$ is the length of the wire and $\Delta V$ is the same regardless of whether $l$ is taken to be at the near or far side of the wire. This can only be true if the electric field is the same at all points across the wire. Therefore, to an excellent approximation, the current is uniformly distributed across the circular cross section of the wire.

## The Wires Aren't Infinitesimally Thin

In the current balance experiment, one usually assumes that the finite diameter of the wires has a negligible effect, and for infinitesimally thin wires separated by a distance $a$ the force is given by

$$F = \frac{\mu_0 L i^2}{2\pi a}. \qquad (5)$$

The currents are treated as if they are concentrated at the centers of the wires.

Of course this is not true; the current density is uniform throughout the cross-sectional area of the wire. In Fig. 2, the current flowing through the upper portion of the top, moveable wire experiences a smaller force (since it's farther away from the source of the magnetic field) than the current flowing through the bottom portion of the top, moveable wire. Are we making a horrible mistake by using the center-to-center distance in our formula for the force? To investigate this we have performed calculations assuming that the bottom, stationary wire has a circular cross section and that the top, moveable wire may or may not have a circular cross section.

The magnitude of the magnetic field created by the stationary wire with a cylindrically symmetric current distribution is given by the expression

$$B(r) = \frac{\mu_0 i}{2\pi r}, \qquad (6)$$

as long as $r$, the distance from the center of the wire, is greater than $b$, the radius of the wire.

To examine the effect of the finite size of the moveable wire, we consider it to have arbitrary cross-sectional area $A$. Again, we assume a uniform current density. We divide the cross section into areas $dA = dxdy$, each of which is the cross section of a filament of current. The current flowing through $dA$ is equal to the current density times the area, which must be $i\left(\frac{dA}{A}\right)$. The center of the moveable wire is located a distance $a$ away from the center of the stationary wire. In the appendix we write the force $dF$ that acts on the filament of current and integrate to obtain the total force acting on the upper wire. It turns out that if the wire has circular cross section, then the force exerted on it by the lower wire is given exactly by Eq. (5).

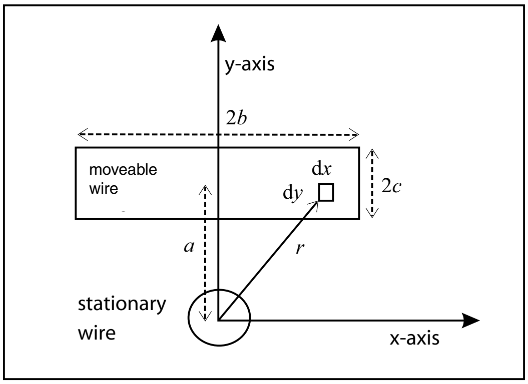


**Fig. 3. The cross-sectional view of the top, moveable wire and a bottom, stationary wire. In this figure, the dimensions for a moveable wire with rectangular cross section are shown.**

## Conclusion

By considering the electrons flowing through the wire and applying geometrical analysis, we have found that the current balance harbors some interesting physics. We have shown that the energy to lift the moveable wire is supplied by the power supply that produces the current. Using integral calculus we have shown that the simplifying assumption that the current is concentrated at the center of the wire (which we know not to be true) does not matter for wires with circular cross sections. Careful consideration of these aspects of the current-carrying wire experiment may enhance any introductory electromagnetism lab course.

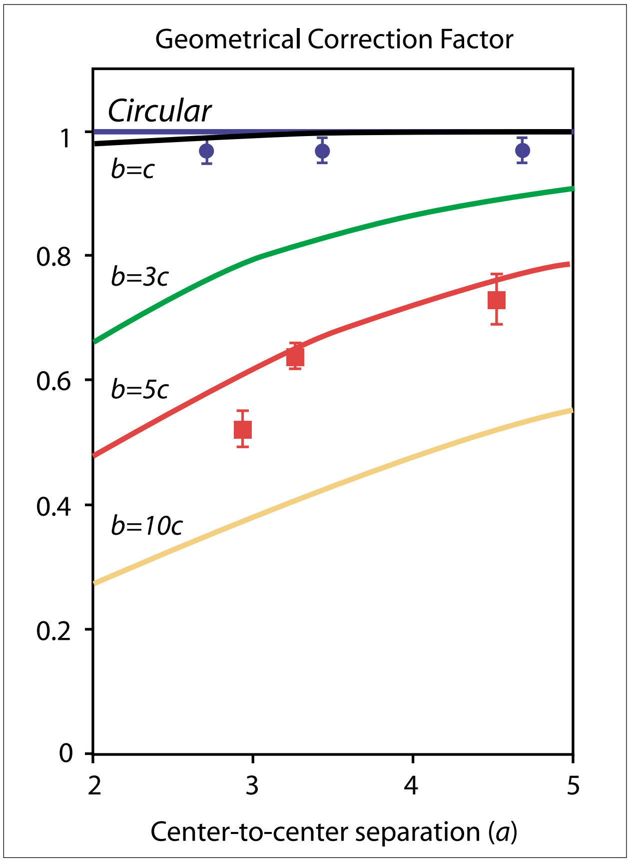


**Fig. 4. The geometrical correction factor, *f*, versus the center-to-center separation (*a*) of the wires. The separation distances are normalized to *c* (one-half the height of the moving wire) in each case. Calculations are shown as solid lines for moveable wires with circular, square, and rectangular cross sections. Data points are shown for a moveable wire with a circular cross section (blue circles) and for a moveable wire with a rectangular cross section that is five times wider than it is high (squares).**


### Acknowledgments

The authors want to thank Clint Cross for his help in fabricating the apparatus used in the experiment and Tom Wakeman for his helpful suggestions. In addition, we thank the referee who directed us to the work described in Ref. 3.


## Appendix

Figure 3 shows a stationary wire having a circular cross section located beneath a moveable wire having cross-sectional area $A$. Both wires carry current $i$. The current flowing through the filament of area $dA = dxdy$ is $i\left(\frac{dA}{A}\right)$. The filament experiences a force $dF$ due to the presence of the stationary wire give by:

$$dF = \left(\frac{\mu_0 Li}{2\pi r}\right)\left(\frac{idA}{A}\right) = \frac{1}{A}\left(\frac{\mu_0 Li^2}{2\pi\sqrt{x^2+y^2}}\right)dxdy. \qquad (7)$$

But what's really of interest to us is the $y$-component of this force (the $x$-component vanishes). By rewriting $dF_y = dF \sin\phi$ in Cartesian components, we obtain an expression that must be integrated over the area $A$:

$$dF_y = \left(\frac{\mu_0 Li^2}{2\pi a}\right)\frac{a}{A}\frac{y}{x^2+y^2}dxdy. \qquad (8)$$

Here we have inserted the center-to-center distance $a$ in the numerator and denominator so that the total force acting on the moveable wire is

$$F = \frac{\mu_0 Li^2}{2\pi a} f\,, \qquad (9)$$

where the dimensionless correction factor $f$ depends on $a$ and the dimensions of the moveable wire. The expression for the total force is just that for two infinitesimally thin wires modified by a correction factor.

In the limit where $a$ is much larger than the dimensions of the wire, the correction factor should be unity. For example, for a moveable wire with a rectangular cross section (with a width $2b$ and a height $2c$, as in Fig. 3), one can show that the correction factor becomes

$$f_{\text{rectangular}} = \frac{a}{4bc} \int_{-b}^{+b} \left\{ \int_{a-c}^{a+c} \frac{y}{x^2+y^2}\, dy \right\} dx. \qquad (10)$$

Carrying out the integration, we obtain.

$$f_{\text{rectangular}} =$$

$$\frac{a}{4bc} \left\{ 2(a+c)\tan^{-1}\left(\frac{b}{a+c}\right) - 2(a-c)\tan^{-1}\left(\frac{b}{a-c}\right) + b\ln\left(\frac{a^2+b^2+c^2+2ac}{a^2+b^2+c^2-2ac}\right) \right\}. \qquad (11)$$

In the limits $a >> b$ and $a >> c$ the first two terms cancel one another. Using L'Hopital's rule on the remaining term, we obtain

$$\lim_{a\to\infty} f_{\text{rectangular}} = 1, \qquad (12)$$

as expected.

Using MathCad[4] to handle the integration, we considered a wire with a rectangular cross section and a circular cross section (with radius $b$). The geometry for the case of the rectangular moveable wire is shown in Fig. 3. The results of the numerical integration are displayed in Fig. 4 with the correction factors plotted as functions of the center-to-center distance for various geometries. Notice that for such a rectangular wire, the correction can be fairly substantial, but for a square wire the effect is only a few percent.

Surprisingly, the correction factor for the circular wire is unity! The relevant integral,

$$f_{\text{circular}} = \frac{a}{\pi b^2} \int_{-b}^{+b} \left\{ \int_{a-\sqrt{b^2-x^2}}^{a+\sqrt{b^2-x^2}} \frac{y}{x^2+y^2}\, dy \right\} dx \text{ ,} \qquad (13)$$

is solved by trigonometric substitutions and integration by parts, and found to be equal to one. This means that for the most common case, of two wires of circular cross section, it's perfectly fine to assume that the wires are infinitesimally thin.

To test our calculations, the correction factor was measured using a standard circular wire and then a rectangular wire manufactured for this purpose. The width of the rectangular wire was five times its height. The data follow the trend of the appropriate curves, although there is some discrepancy for the case of the rectangular wire with the smallest separation distance.

## References


1. For example, the Current Balance, WL2353, Sargent-Welch, P.O. Box 5229, Buffalo Grove, IL 60089-5229; http://www.sargentwelch.com.
2. R.A. Nelson, "Foundations of the international system of units (SI)," *Phys. Teach.* **19**, 596–613 (Dec. 1981).
3. Eugene P. Mosca, "Magnetic forces doing work?" *Am. J. Phys*. **42**, 295–297 (1974).
4. MathCad 2001 Professional, MathSoft, Inc. © 1986-2000; http://www.mathsoft.com.


PACS: 01.50P, 41.01A, 41.90


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