Hour Exam 2 Solutions

Math 384

April 2015

Question 1

Find a PDF on $[0,1]$ such that $P(X = x) \propto x^{2.5}$. From the problem statement the PDF must be of the form $p(x) = kx^{2.5}$, and to be a PDF it must integrate to 1 over the interval $[0,1]$, which allows you to find k:

$$\begin{aligned} 1 &= \int_0^1 kx^{2.5} dx \\ &= k\left[\frac{x^{3.5}}{3.5}\right]_0^1 \\ &= k\left[\frac{1}{\frac{7}{2}}\right] \\ &= \frac{2}{7}k \end{aligned}$$

Thus $k = \frac{7}{2}$, and so $p(x) = \frac{7}{2}x^{2.5}$.

Question 2

Equation for Phong specular highlights in an alternate universe. Phong's basic equation is $I_o = k_s \cos^a \Theta I_i$ where I_o is the reflected light intensity and I_i is the incoming light intensity. Assuming unit-length vectors, $\cos\Theta = \vec{R} \cdot \vec{V} = \vec{L} \cdot \vec{V}$. So the final equation is $I_o = k_s(\vec{L} \cdot \vec{V})^a I_i$.

Question 3

How does a vector from a point to a light in canonical coordinates transform to world coordinates? Let $\vec{P}'$ and $\vec{S}'$ be the point and light position in world coordinates, respectively, so $\vec{P}' = T\vec{P}$ and $\vec{S}' = T\vec{S}$. Now let $\vec{L}'$ be the world-coordinate vector from $\vec{P}'$ to $\vec{S}'$. Then ...

$$\begin{aligned} \vec{L}' &= \vec{S}' - \vec{P}' \\ &= T\vec{S} - T\vec{P} \\ &= T(\vec{S} - \vec{P}) \\ &= T\vec{L} \end{aligned}$$

Question 4

Phineas's pseudocode modified to estimate the integral by using 32 samples could look like this:

```
microwaveIllumination = 0
for i = 1 : 32
    theta = rand * π
    phi = rand * 2 * π
    probability = 1 / ( 2 * π * π )
    directionX = sin(theta) * cos(phi)
    directionY = cos(theta)
    directionZ = sin(theta) * sin(phi)
    direction = vector( directionX, directionY, directionZ )
    sampleRay = ray( P, direction )
    t = trace( sampleRay )
    if t >= inf
      microwaveIllumination = microwaveIllumination + cmb(theta,phi) / ( probability * 32 )
    end
end
```