

Modeling Population Growth Using Life History Data

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Point = to use age structure data to make more accurate predictions of population change over time.

$$A \times \vec{N}_t = \vec{N}_{t+1}$$

$$A \times N_t = N_{t+1}$$

$$\begin{bmatrix} \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \phantom{N_{t+1}} \\ \phantom{N_{t+1}} \\ \phantom{N_{t+1}} \end{bmatrix}$$

$$\lambda \times N_t = N_{t+1}$$

$$A \times \vec{N}_t = \vec{N}_{t+1}$$

Probability of
surviving to next
Time step

\underline{x} (age)	\underline{l}_x	\underline{b}_x	\underline{P}_x
0	1.0	0	
1	0.8	2	0.8/1.0
2	0.4	3	
3	0.1	1	
4	0	0	

$$P_x = l_{x+1}/l_x$$

$$A \times \vec{N}_t = \vec{N}_{t+1}$$

x (age)	l_x	b_x	P_x	F_x
0	1.0	0		
1	0.8	2	0.8	1.6
2	0.4	3	0.5	1.5
3	0.1	1	0.25	0.25
4	0	0	0	0

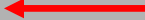
Probability of surviving to next time step

Age-specific fecundity

$$P_x = l_{x+1}/l_x$$

$$F_x = b_x * P_x$$

Next Year's Population:

$$\begin{matrix}
 \boxed{A} & \times & \vec{N}_t & = & \vec{N}_{t+1} \\
 \begin{bmatrix} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{bmatrix} & \times & \begin{bmatrix} N_0 \\ N_1 \\ N_2 \\ N_3 \end{bmatrix} & = & \begin{bmatrix} \\ \\ \\ \end{bmatrix}
 \end{matrix}$$


Multiplying a matrix by a vector

Next Year's Population:

$$\begin{array}{c}
 \boxed{A} \quad \times \quad N_t = N_{t+1} \\
 \left[\begin{array}{cccc} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{array} \right] \times \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 167 \\ \\ \\ \end{bmatrix}
 \end{array}$$

Next year's babies =

$$\begin{aligned}
 N_{0,t+1} &= F_0 N_{0,t} + F_1 N_{1,t} + F_2 N_{2,t} + F_3 N_{3,t} \\
 &= 80 + 75 + 12.5 + 0
 \end{aligned}$$

Next Year's Population:

$$\begin{array}{c}
 \boxed{A} \quad \times \quad N_t = N_{t+1} \\
 \left[\begin{array}{cccc} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{array} \right] \times \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 167 \\ 40 \\ \\ \end{bmatrix}
 \end{array}$$

Next Year's Population:

$$\begin{array}{c} \boxed{A} \quad \times \quad N_t \quad = \quad N_{t+1} \\ \left[\begin{array}{cccc} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{array} \right] \times \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 167 \\ 40 \\ 25 \end{bmatrix} \end{array}$$

Next Year's Population:

$$\begin{array}{c} \boxed{A} \quad \times \quad N_t \quad = \quad N_{t+1} \\ \left[\begin{array}{cccc} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{array} \right] \times \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 167 \\ 40 \\ 25 \\ 12.5 \end{bmatrix} \end{array}$$

Next Year's Population:

$$\begin{matrix} & \text{A} & & \times & N_t & = & N_{t+1} \\ \begin{bmatrix} 1.6 & 1.5 & 0.25 & 0 \\ & 0.8 & & \\ & & 0.5 & \\ & & & 0.25 \end{bmatrix} & & & \times & \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} & = & \begin{bmatrix} 167 \\ 40 \\ 25 \\ 12.5 \end{bmatrix} \end{matrix}$$



Next Year's Population:

$$\begin{matrix} & \text{A} & & \times & N_t & = & N_{t+1} \\ \begin{bmatrix} 1.6 & 1.5 & 0.25 & 0 \\ & 0.8 & & \\ & & 0.5 & \\ & & & 0.25 \end{bmatrix} & & & \times & \begin{bmatrix} 50 \\ 50 \\ 50 \\ 50 \end{bmatrix} & = & \begin{bmatrix} 167 \\ 40 \\ 25 \\ 12.5 \end{bmatrix} \end{matrix}$$

All other values in A are 0

Next Year's Population:

$$\begin{array}{c}
 \boxed{A} \quad \times \quad N_t = N_{t+1} \\
 \left[\begin{array}{cccc} 1.6 & 1.5 & 0.25 & 0 \\ 0.8 & & & \\ & 0.5 & & \\ & & 0.25 & \end{array} \right] \times \left[\begin{array}{c} 50 \\ 50 \\ 50 \\ 50 \\ 200 \end{array} \right] = \left[\begin{array}{c} 167 \\ 40 \\ 25 \\ 12.5 \\ 245 \end{array} \right]
 \end{array}$$

A=				
	1.6	1.5	0.25	0
	0.8	0	0	0
	0	0.5	0	0
	0	0	0.25	0
Nt				
	50			
	50			
	50			
	50			
	200			