

Math 233

Complete the following exercises.

1. Find $\ker(T)$ of $T : M_{2 \times 2} \longrightarrow M_{2 \times 2}$ is the linear transformation defined by

$$T \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} c - 2a & 2b \\ 0 & a - d + c \end{pmatrix}.$$

2. Consider the set of 2×2 matrices

$$S = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 2 \\ 0 & 0 \end{pmatrix} \right\}.$$

- (a) Is S a linearly independent set?
 - (b) Find a basis \mathcal{B} for $\text{span}(S)$.
 - (c) What is the dimension of $\text{span}(S)$?
 - (d) Write the matrix $A = \begin{pmatrix} 6 & 2 \\ 0 & 1 \end{pmatrix}$ as a linear combination of the vectors in the basis \mathcal{B} .
 - (e) What is the coordinate vector $[A]_{\mathcal{B}}$?
 - (f) For what value of k is $[A]_{\mathcal{B}} \in \mathbb{R}^k$?
 - (g) Let \mathcal{C} be the standard basis of $M_{2 \times 2}$. For what value of k is $[A]_{\mathcal{C}} \in \mathbb{R}^k$? What is $[A]_{\mathcal{C}}$?
3. Let V be a subspace of \mathbb{P} of polynomials of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$ such that $a - c = 0$ and $e - 3b = 0$.
- (a) Find a basis for V .
 - (b) What is the dimension of V ?