Math 239: Intro to Mathematical Proof

Instructor: Aaron Heap Office: South 330C Web Page: <u>http://www.geneseo.edu/math/heap</u> Textbook: <u>ProofSpace</u> E-mail: <u>heap@geneseo.edu</u> Phone: 245-5391

Course Info: We will cover various techniques of mathematical proof, logic, set theory, functions, and equivalence relations. Topics are subject to change depending on the progress of the class, and various topics may be skipped due to time constraints. We will also have the option to add certain topics based on student interest.

For many of you, this will be the first mathematics course that uses a more mathematically sophisticated approach than that found in your standard calculus courses. At this point in your mathematical careers, you are mostly familiar with *computational* mathematics but have had very little exposure to *theoretical* and *proofbased* mathematics. This is one of the first courses where you will be asked to write an argument in order to solve a problem. That is, you will have to write "proofs". Most of you may have some basic experience with this through your high school geometry course. Others may have some experience from Linear Algebra. In this course, we will be learning a variety of proof techniques, and everyone should be comfortable with the process by the end of the semester.

This course is known as a "transition course." That is, the purpose of the course is to bridge the theoretical gap between lower-level, computation-based math classes and upper-level, proof-based math classes. The topics and tools learned here will permeate the remainder of your mathematical education and will provide you with the necessary skills to become successful mathematicians. Our primary purpose in this class is to help you make progress in developing analytical, critical reasoning, problem solving, and communication skills and acquiring mathematical habits of mind. The number of topics we will cover is deliberately kept to a minimum so that you can concentrate on developing careful use of mathematical language, practice logical reasoning skills, and learn theorem-proving skills. The emphasis throughout the course is on process rather than on content. Moreover, work with written and oral communication of mathematical ideas and independent reading of mathematical content are essential parts of this course.

Learning Outcomes: Upon successful completion of this course, a student will be able to:

- Apply the logical structure of proofs and work symbolically with connectives and quantifiers to produce logically valid, correct and clear arguments,
- Perform set operations on finite and infinite collections of sets and be familiar with properties of set operations,
- Determine equivalence relations on sets and equivalence classes,
- Work with functions and in particular bijections, direct and inverse images and inverse functions,
- Construct direct and indirect proofs and proofs by induction and determine the appropriateness of each type in a particular setting. Analyze and critique proofs with respect to logic and correctness,
- Unravel abstract definitions, create intuition-forming examples or counterexamples, and prove conjectures.

Grading: Your grade in this course will be based entirely on your attainment of the skills described in the course learning outcomes above. There will be regular problem sets, two midterm exams, and a final exam. Your overall grade will be determined as shown below. More details and exam dates can be found on the course website. Exams will be given during class. Exams are closed book, closed notes, closed friends, and open brain. Calculators, phones, and other electronic devices will NOT be permitted in exams. Class participation will be based on your willingness to ASK and ANSWER questions in class. There will be active discussion, and you will also be required to present some proofs to the class. It is imperative that you keep up with the assignments. This will help you answer my questions and help you ask more essential, thought-provoking questions during class.

In all written work, you must show your work and clearly show the process and reasoning you went through in order to solve the problem. The problems I work for you in class will provide good examples of how your homework and exam problems should be written up. All assessment will be based on your ability to communicate a correct solution and explain your reasoning. It is absolutely essential to write clearly and completely. It is your responsibility to write in a way that tells me that you understand the problem and its solution.

10% - Quizzes and class participation 15% - Evaluated problem sets 25% - Exam 1 25% - Exam 2	A : 93-100 A- : 90-92 B+ : 87-89 B : 83-86	C+: 77-79 C : 73-76 C-: 70-72 D : 60-69
25% - Exam 2	B : 83-86	D : 60-69
25% - Final exam	B- : 80-82	E : 0-59

Course Format: This course is guided by *formative assessment* – in plain terms, you tell us what you know and what you do not know, and we try to help you gain the skills you need accordingly. We will be using a learning technique called "flipping the classroom." In this format, you will *acquire* the majority of your knowledge outside of class through video lectures and follow-up comprehension problems. Then, you will *apply* your knowledge to high-level skills in the classroom. This format has several advantages, including:

- The instructional team are available when the students most need us; rather than me delivering lectures and then you trying difficult problems by yourself at home, I am here when you are trying those difficult problems and are most likely to ask for help.
- Class time is used more actively.
- The instructional team gets to know you better and cater to your individual needs.
- We can focus more on high-level problems that will help you be more successful throughout your mathematical career.

Most importantly, we have tried this format before and found that in this format, we are better able to adjust the course to each individual's learning style. It is important to note that this course format should be neither more nor less work than a traditional class. It is just a matter of when you are doing what. For obvious reasons, attendance, though not factored in your grade, is vital to your success as a student of mathematics.

We will mostly be working out of video lectures and assignments created specifically for our course. To access these resources, go to the course website or go to http://go.geneseo.edu/proofspace. Each problem set comes with a list of supplemental problems. If you are having trouble understanding a concept, we encourage you to refer to our supplementary textbook, which is *Mathematical Reasoning: Writing and Proof* by Ted Sundstrom. This text follows roughly the same chronology as our course. It is an excellent resource for extra problems and alternative viewpoints. Dr. Sundstrom has generously made the book available for free online at http://scholarworks.gvsu.edu/books/7/. If you choose to download it, we encourage you to e-mail him a word of thanks. The problem numbers in the list of supplemental problems refer to the exercises in the online version. Although the textbook is of the highest quality, when working with definitions in this course, **always refer to the definition given in our videos**. The differences between our delivery and Sundstrom's are few, but they are important and deliberate.

"Homework" consists of two separate things: Reading/Video Assignments and Problem Sets. There will be regular homework assignments that must be completed before class on the due date. Typically, the reading/video assignments and quizzes will be daily, and problem sets will be due weekly. See the website for the homework requirements and schedule. Please note that you are **STRONGLY ENCOURAGED** to **WORK IN GROUPS** of 2 or 3 students. However, each student is responsible for writing up his or her own homework. Please use whatever resources aid you in learning the material, including me, other students, professors, other math books, etc. **I strongly recommend that you avoid copying from classmates, textbooks, or the internet. This is known as plagiarism, and it is considered cheating.**

Day-to-Day Schedule: Almost every day, you will watch some short videos (usually less than 30 total minutes) and try a few problems online *before class*. We will look at how the class as a whole did on each problem and address important issues in a short discussion period at the start of class. We will also discuss large-scale questions as a class.

Then, we will split into pairs to start working on a handful of problems that will be due the following week. At this time, we can address individual questions. After class, you will finish the assignments and turn them in. You will get them back shortly thereafter with a numerical grade and feedback. Use this as a springboard to meet with us about your progress and prepare for the comprehensive exams.

Watching the screencasts and trying the problems are required, NOT optional. Your chances of passing this course are infinitesimally small unless you do this. Because this will form the foundation of your learning for this class, it is HIGHLY RECOMMENDED that you take notes on what you read or watch. In addition to standard note taking, it may be helpful to keep a separate list of definitions learned and a separate list of theorems learned. These will be the tools you use throughout the semester on problem sets and exams.

Each problem set is broken up into four sections:

- 1. Discussed
- 2. Evaluated
- 3. Supplementary
- 4. Advanced

Before all of these, however, is the **comprehension quiz**. These will be completed on <u>WeBWorK</u> **BEFORE** you come to class. You must understand the material assessed on the quiz before you can begin the problem set.

The **discussion problems** are few in number, but we will discuss them as a class. Although you are required to complete these, they will NOT be handed in to be graded. The **evaluated problems** will most likely be completed independently, out of class, and are sometimes easier or harder than the discussed problems; they are often related to the corresponding in-class problem. These are the only problems on the problem set that will be handed in and graded.

If you would like more practice or need to review, reference the **supplementary problems**. Each refers to the end-of-section problems in Dr. Sundstrom's book. These are problems that have been assigned in this course in previous semesters.

The **advanced problems** are excellent choices for students who are comfortable with the material and would like to explore the topics more.

The problem sets were very thoughtfully created. No problem should be considered trivial or a waste of time. If there is even one problem you do not understand excellently by the end of our chapter, you should come talk out the issue with us.

Extra Help: There are two obvious ways for you to get some help in this class. Ask a classmate or ask your professor. I am willing to spend a few minutes in class answering questions about homework problems. However, if you have many questions, I recommend taking advantage of my office hours. I will say it again...USE MY OFFICE HOURS! My job is to help you -- come to office hours even if you have just a small question. Do not wait until you get too far behind. If my office hours are not convenient for you, make an appointment by sending an email or asking after class. Please come see me as soon as you feel lost -- it is important that I know how you are doing so I can adjust the level of the class if necessary. I <u>WANT</u> to help you, and I <u>WANT</u> everyone to do well.

Don't let yourself get too far behind! I would be happy to see you if you feel you need some assistance. Come and get your questions cleared up right away.

Accommodations: SUNY Geneseo will make reasonable accommodations for persons with documented physical, emotional, or cognitive disabilities. Accommodations will be made for medical conditions related to pregnancy or parenting. Requests for accommodations including letters or review of existing accommodations should be directed to the Office of Disability Services in Erwin Hall 22 or <u>disabilityservices@geneseo.edu</u> or 585-245-5112. Students with letters of accommodations should submit a letter to each faculty member at the beginning of the semester and discuss specific arrangements. Additional information on the Office of Disability Services is available at <u>www.geneseo.edu/dean_office/disability_services</u>.