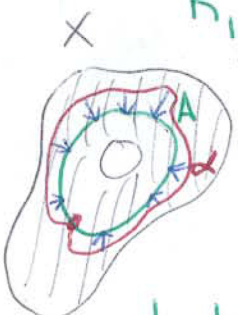
<u>Thm</u> Suppose A is a deformation retract of X. Then $\pi_1(X) \cong \pi_1(A)$.

<u>proof</u>: Let R be the deform. retraction from X to A, and $R_1(x) = R(x,1) \in A$.

Let $i : A \to X$ be inclusion, $i(a) = a$ $\forall a \in A$.

The induced map $i_* : \pi_1(A) \to \pi_1(X)$ is a homomorphism since i is cont.

<u>Onto</u>: Let α be a loop in X. Then $R_1 \circ \alpha$ is a loop in A, and the retraction R describes a homotopy from α to $R_1 \circ \alpha$



$$\Rightarrow \quad i_*(\langle R_1 \circ \alpha \rangle) = \langle R_1 \circ \alpha \rangle = \langle \alpha \rangle$$

$$\Longrightarrow \quad i_* \text{ is onto}$$

<u>1-1</u>: Suppose α, β are loops <u>in A</u> s.t. $\langle \alpha \rangle = \langle \beta \rangle$ <u>in X</u>. Let H be a homotopy from α to β <u>in X</u>. Then $R_1 \circ H$ is a homotopy from α to β <u>in A</u>.

$$i_*(\langle \alpha \rangle) = i_*(\langle \beta \rangle) \Rightarrow \langle \alpha \rangle = \langle \beta \rangle \text{ in } \pi_1(X)$$

$$\Rightarrow \langle \alpha \rangle = \langle \beta \rangle \text{ in } \pi_1(A)$$

$$\Rightarrow i_* \text{ is 1-1.}$$

$\therefore$ i_* is an isomorphism. ∎