Algebraic Topology - Homework 3

Problem 1. Prove that the number of path components is a topological invariant.

Problem 2. Using our original formal definition of the n-sphere S^n, determine the path components of $\mathbb{R}^{n+1} - S^n$ for $n \geq 0$.

Problem 3. Write the upper-case letters of the alphabet using line segments and arcs. Which of the resulting topological spaces are homeomorphic?

Problem 4. Show that the intervals $[0, 1)$ and $[0, 1]$ are not homeomorphic.

Problem 5. Show that the intervals $[0, 1]$ is not homeomorphic to the square region $[0, 1] \times [0, 1]$.

Problem 6. Show that the double cone $\{(x, y, z) \mid x^2 + y^2 = z^2\}$ is not homeomorphic to $\mathbb{R}^2$.

Problem 7. Show that the torus is not homeomorphic to a genus 2 surface.

Review of Groups:

Problem 8. Using your notes from Math 330 or an abstract algebra textbook, find 3 interesting/useful/helpful theorems or properties of cyclic groups. Discuss them with your partner, and explain to them why you think they are interesting/useful/helpful. Please note that I'm actually hoping you'll take this question seriously.

Problem 9. How many different groups, up to isomorphism, are there of order 1? 2? 3? 4? 5? 6? 7? 8? 9? 10? What are they? Which ones are abelian? Try to do this without just looking up a table of groups. If you are able, explain why these are the only groups of each order. HINT: Only groups of order 8 should pose a challenge.