

Algebraic Topology - Homework 7

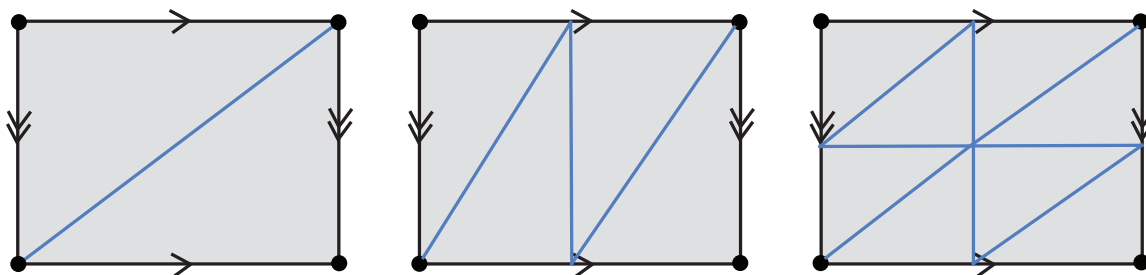
Problem 1. Go to Wikipedia and read about the “Poincaré Conjecture” and “Homotopy groups of spheres.”

Problem 2. Let G be any group, and let G' be its commutator subgroup. The **abelianization** of G is the quotient group G/G' .

- (a) Easy: Is G/G' a group? (Yes) Explain. (Use one sentence.)
- (b) Is G/G' abelian? Explain.
- (c) What is the abelianization of S_3 ?
- (d) If G and H are isomorphic, then their abelianizations are too. Is the converse true?
- (e) Let X_g be an orientable surface of genus g . What is the abelianization $\pi_1(X_g)$?
- (f) Conclude that X_g is not homeomorphic to X_h if $g \neq h$.

Problem 3. How many faces does an n -simplex have?

Problem 4. Here are three simplicial complexes whose underlying topological space is the torus. Determine if these are triangulations of the torus.



Problem 5. Modify the triangulation of the torus given in class to produce a triangulation of the Klein Bottle.

Problem 6. Give a triangulation of the projective plane and the dunce cap.