Algebraic Topology - Homework 8

Problem 1. Consider the oriented 3-simplex $\sigma^3 = \langle v_0 v_2 v_1 v_3 \rangle$.

(a) Without finding them or counting them, how many total orderings of these vertices are equivalent to this orientation? Explain.

(b) What are the induced orientations on all of the faces of σ^3, i.e. the n-simplexes with $n = 0, 1, 2$?

(c) Given the oriented n-simplex $\sigma^n = \langle v_0 v_1 \dots v_n \rangle$, how many total orderings of these vertices are equivalent to this orientation?

Problem 2. Let σ be the 2-simplex with orientation given by $\langle v_0 v_1 v_2 \rangle$.

(a) Draw σ with the orientation on each face induced by the given orientation.

(b) Compute $\partial_2(\sigma)$.

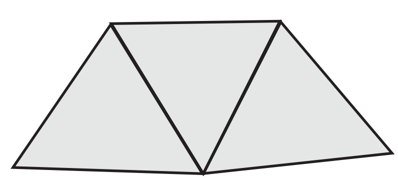
(c) Give the other two equivalent orientations of σ and compute ∂_2 of them. Do your answers agree? Why is this important?

(d) Compute $\partial_1(\partial_2(\sigma))$.

Problem 3. Let K be the simplicial complex pictured below.

(a) Pick an orientation on only one of the 2-simplexes of K. Using that choice, give an induced orientation on all remaining simplexes so that the orientations are compatible. (This is also sometimes called a ***coherent orientation***.) As you obtained your induced orientation, did you get to make any choices or was there only one option of orientation for any given simplex?

(b) Using the orientation from (a), compute the homology groups $H_p(K)$ for all $p \geq 0$.



Problem 4. (a) Give a triangulation of an annulus.

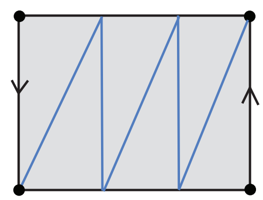
(b) Pick an orientation on only one of the 2-simplexes, and find a coherent orientation for the remaining simplexes in the triangulation from (a).

(c) Compute all of the homology groups of the annulus using the orientation from (b).

Problem 5. Below is a triangulation of the Mobius strip M. To make your life easier, let the arrows that indicate the gluing of the sides also be the orientation on that 1-simplex.

(a) Show that it is not possible to find a coherent orientation. This means that the Mobius strip is ***nonorientable***.

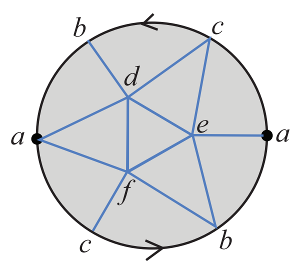
(b) Pick an orientation for each simplex of this triangulation. (Of course, it will not be a *coherent* orientation.) Use this orientation to compute the homology groups $H_p(M)$ for all $p \geq 0$.



Problem 6. Below is a triangulation of the projective plane P. (The letters just indicate the vertices.)

(a) Use the triangulation to determine if P is orientable or nonorientable.

(b) Pick an orientation for each simplex of this triangulation. (It may or may not be a *coherent* orientation.) Use this orientation to compute the homology groups $H_p(P)$ for all $p \geq 0$.



Problem 7. Below on the left is an oriented triangulation of the Klein bottle K. The letters indicate the vertices, and the orientations are indicated in red. On the right is a Δ-complex of the Klein bottle, with edges labeled with x, y, and z.

(a) Is the orientation on the triangulation coherent?

(b) Using either the simplicial complex on the left or the Δ-complex on the right, compute the homology groups $H_p(K)$ for all $p \geq 0$.