Honors Math III - Practice Exam 1

(Note that the exam will NOT be this long.)

1 Definitions

1. (0 points) Let U be a subset of a vector space V. Let $S = \{v_1, v_2, \ldots, v_n\}$ be another subset of V.

(a) Define "U is a **subspace** of V".

(b) Define "S is **linearly independent**".

(c) Define "S **generates** V".

2 Vector Spaces and Subspaces

2. (0 points)

(a) Give three examples of 4-dimensional vector spaces.

(b) Give one example of an infinite dimensional vector space.

(c) Give an example of a zero-dimensional vector space.

3. (0 points) Let S_1 and S_2 be subspaces of a vector space V. Prove that the union $S_1 \cup S_2$ is a subspace of V if and only if one is contained in the other (that is, either $S_1 \subseteq S_2$ or $S_2 \subseteq S_1$.)

3 Linear Independence, Generating Sets, and Bases

4. (0 points) Let $S = \{x^2 + 3x, x - 2\}$ be a subset of $P_2(\mathbb{R})$.

(a) Explain why S is *not* a basis of $P_2(\mathbb{R})$.

(b) Is $\frac{1}{3}x^2 + 2$ in $\operatorname{span}(S)$? Explain.

(c) Is $2x^2 + 5x + 4$ in $\operatorname{span}(S)$? Explain.

5. (0 points) Consider the 3 vectors in $\mathbb{R}^3$ given by $v_1 = (1, 1, -1)$, $v_2 = (1, 1, 1)$, and $v_3 = (3, 5, 7)$. Decide whether these 3 vectors provide a basis for $\mathbb{R}^3$. Justify your answer.

6. (0 points) Let W be the subspace of $\mathbb{R}^3$ given by

$$W = \{(x, y, z) \mid x + y + z = 0 \text{ and } x - y - z = 0\}.$$

Find a basis for W and the dimension of W.

7. (0 points) Let $S = \{v_1, v_2, \ldots, v_n\}$ be a set of n vectors in a vector space V. Show that if S is linearly independent and the dimension of V is n, then S is a basis of V.

8. (0 points) Consider the subset $S = \{x^3 - 2x^2 + 1, 4x^2 - x + 3, 3x - 2\}$ of $P_3(\mathbb{R})$.

(a) Explain how you know that S does not generate $P_3(\mathbb{R})$.

(b) Can you add a vector v to S so that $S \cup \{v\}$ is a basis of $P_3(\mathbb{R})$? Justify and find such a vector if possible.

9. (0 points) Let V be a vector space over $\mathbb{R}$, and let $x, y, z \in V$. Prove that $\{x, y, z\}$ is linearly independent if and only if $\{x + y, y + z, z + x\}$ is linearly independent.

10. (0 points) Consider the vector space $V = P_1(\mathbb{R})$.

(a) Explain why you know that the set $\beta = \{1 + x, 1 - 2x\}$ is a basis of V.

(b) Express $p(x) = 2x - 3$ as a linear combination of β.