Math 388

Homework #12

For Problems 1-3, do not work with anyone. The items listed should be topics learned in this class. List as many things as you can. The point of these exercises is to help you understand how most of the things we have learned are related.

1. Let V and W be vector spaces of dimension n and p, respectively. Let $T : V \to W$ be a linear transformation. Compile a list of statements that are equivalent to the statement: **"T is injective."** Also mention a restriction of the value of p relative to the value of n.

2. Let V and W be vector spaces of dimension n and p, respectively. Let $T : V \to W$ be a linear transformation. Compile a list of statements that are equivalent to the statement: **"T is surjective."** Also mention a restriction of the value of p relative to the value of n.

3. Let V and W be vector spaces of dimension n and p, respectively. Let $T : V \to W$ be a linear transformation. Compile a list of statements that are equivalent to the statement: **"T is invertible."** Also mention a restriction of the value of p relative to the value of n.

4. Consider the matrix
$$A = \begin{pmatrix} -1 & 2 & -3 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

   (a) Show that A is diagonalizable by finding eigenvalues and eigenvectors of A.

   (b) Find an invertible matrix P and diagonal matrix D such that $A = PDP^{-1}$.

   (c) Compute A^k for an arbitrary positive integer k.