Math 388

Homework #13

1. Find the characteristic polynomial of each of the following differential equations. Also rewrite each differential equation as a system of first-order linear differential equations and as a matrix equation $\vec{x}\,' = A\vec{x}$. Compare the characteristic polynomial of the original differential equation with the characteristic polynomial of the matrix A.

   (a) $ay'' + by' + cy = 0$

   (b) $ay''' + by'' + cy' + dy = 0$

2. From Section 18.4 of the textbook, do problems 1, 2, 3, 4(a,b), 5

3. Consider the matrix

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{pmatrix}$$

   (a) Using the definition of the exponential of a matrix, write out the first 4 terms of e^A.

   (b) Since A is diagonalizable, write e^A in closed form (i.e. as a single matrix whose entries are NOT an infinite sum.)

   (Hint: In the previous homework assignment you found the eigenvalues and eigenvectors of A, expressed it as $A = PDP^{-1}$, and used this to find A^k.)

4. Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

   (a) If you use the definition of the exponential of a matrix, e^{At} is an infinite series. Use the technique we learned in class to express e^{At} in closed form.

   (b) Find the solution to $\vec{x}\,' = A\vec{x}$ with $\vec{x}(0) = (1, 2, 3, 4)$.

   (c) Find a basis for the solution space of $\vec{x}\,' = A\vec{x}$.