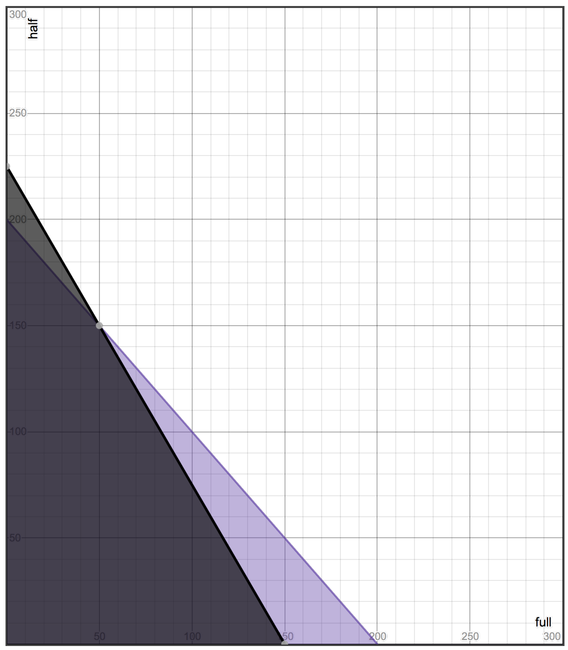
Finite Mathematics Problem Set I Solutions

PSJ

7.3 Solve the linear programming problem formulated in 7.1 13 for the last assignment.

That is at the top of this document. Start by graphing the feasible region (I have put full teams on the horizontal axis):

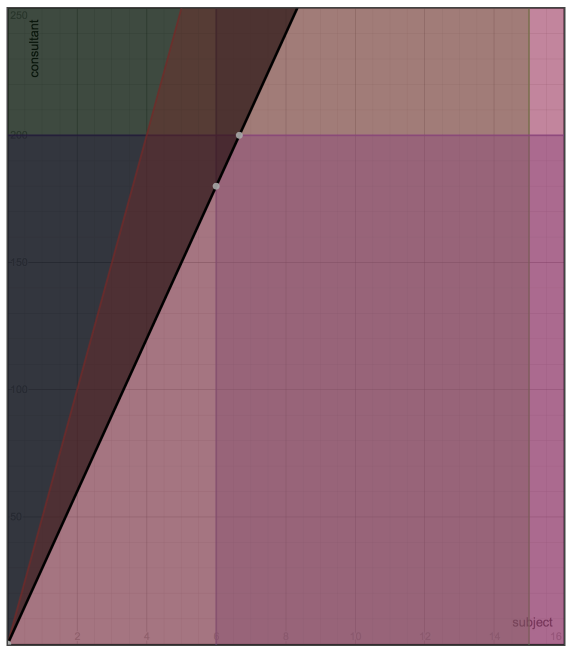


The four corners of the region are $(0, 0)$, $(0, 200)$, $(50, 150)$ and $(150, 0)$. At these corner points, the number of inoculations are, respectively, $0, 20000, 24000$, and 27000. This is an interesting result. It says we should make all full teams, and tell the extra doctors we don't need them (because there will be 150 full teams and 50 leftover doctors. Nurses do all the work - they make all the difference. Interesting.

7.3 11. A psychologist plans to conduct an experiment which involves subject who perform activities. After data have been collected, the data are to be analyzed by a team of expert consultants. The psychologist has 15 subject hours available, and they will need to use at least 6 of them. They have funds for a maximum of 200 minutes of consultant time, and each hour of subject time requires at least 30 minutes of consultant time to analyze the data. Depending upon the depth of the analysis, up to 50 minutes of consultant time per subject hour can be profitably used. The information which the psychologist obtains from the experiment depends upon the number of subject hours and the amount of analysis. They estimate that, in appropriate units, 1 unit of information is obtained from each subject hour and 1 unit is obtained from each 25 minutes of consultant analysis. How should the experiment be organized (i.e. how many subject hours and how much consultant analysis) to give the maximum information?

And now we do all the work for one problem from the beginning. The two variables are subject (in hours), s, and consultant (in minutes), c. As always we have $s, c \geq 0$. But, in fact we have $6 \leq s \leq 15$. We also have $c \leq 200$, and $30s \leq c \leq 50s$. The objective function is $I = s + \frac{c}{25}$.

Here is a graph of the feasible region (I have subject hours on the horizontal axis):



There's a lot going on there, but the feasible region is the rather tiny triangle with two vertices marked wth grey points. There are many constraints and they are difficult to all manage, so there is just barely an overlap. In the end, this makes our work easier. The corner points are $(6, 180)$, $(6, 200)$, and $(6\frac{2}{3}, 200)$ i.e. $(6 \text{ hours } 40 \text{ minutes}, 200)$.

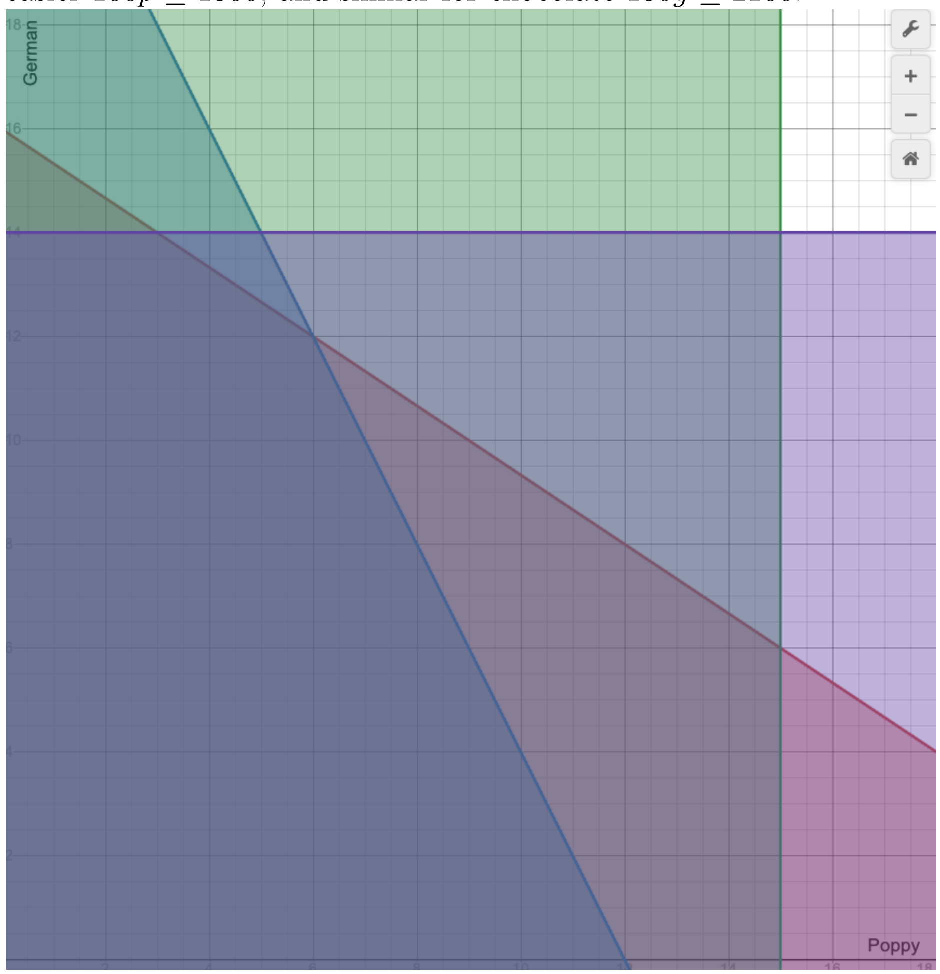
This time the consultants turned out to be pretty important, as that 200 minute limit was rather limiting. The information for these three situations respectively are $13\frac{4}{5}$, 14, and $14\frac{2}{3}$. Unsurprisingly they are close to the same, but the $(6 \text{ hours } 40 \text{ minutes}, 200)$ point is a little better than the others.

7.3.29. A bakery makes two types of cakes each day: poppy seed and German chocolate. The profit to the bakery is \$2 on each poppy seed cake and \$4 on each German chocolate cake. A poppy seed cake requires 400g of flour, 200g of butter, and 100g of poppy seeds. A German chocolate cake requires 600g of flour, 100g of butter, and 150g of chocolate. There are 9600g of flour, 2400g of butter, 1500g of poppy seeds, and 2100g of chocolate.

a. How many cakes of each type should be made to yield maximum profit? What is the maximum profit?
b. When the baker produces cakes to yield maximum profit, does any flour remain unused at the end of the day? How much?
c. Repeat b. for butter.

Here we go again. The variables are p for number of poppy seed cakes, and g for number of German chocolate cakes. We want to maximise profit $= 2p+4g$. As typical, we can't make negative of either $p \geq 0$, $g \geq 0$. We have constraints each about flour, butter, poppy seeds, and chocolate. Here's the one for flour: $400p + 600g \leq 9600$. One for butter: $200p + 100g \leq 2400$. One for seeds is easier $100p \leq 1500$, and similar for chocolate $150g \leq 2100$.

Here is the region:



There are 5 corners. (0,0) (0,14), (3,14), (6,12) and (12,0).

So, we need to check the profit at each. The first one is the minimum of zero. The remaining ones are \$56, \$62, \$60 and \$24. So the best one is at (3,14), i.e. 3 poppy seed cakes and 14 German cakes (makes sense since they are more profitable). The maximum profit is \$62.

In my graph, the red line is for flour. Since the point (3,14) is on the red line, flour is used up. We can check this $400(3) + 600(14) = 9600$. The blue line is for butter, but (3,14) is not on the blue line. Butter is not used up, we only use $200(3) + 100(14) = 2000$g of the 2400g total. We have 400g left.