222 §7.3

Not as much lab wrap up this time. Because we'll do the same things from a different perspective. It is worth noticing something: e is a number ((1) So, what is $\frac{d}{dx}e^2$?) and $e \simeq 2.718281828459045$.

We defined $\ln(x) = \int_1^x \frac{1}{t}dt$. Now we *define* e^x to be the inverse function for $\ln(x)$ (which is increasing, so it has an inverse). So, by definition of inverse $e^{\ln x} = x$ and $\ln(e^x) = x$. By implicit differentiation of the second equation we can discover the derivative of e^x. (2) Do this.

From this it should be quick to also get (3) $\int e^x dx$

As practice, (4) integrate and differentiate $e^{\pi x}$. (5) differentiate e^{e^x} and (6) integrate $\frac{e^{2x}-e^{4x}}{e^x}$ and (7) xe^{-4x^2}.

So far we have only worked with natural logarithms and exponentials, base e. What if we use other bases? Well, it takes a little work, because calculus works well with naturals (just like it works well with radians, as a reminder (8) if x is in degrees, what is the derivative of $\sin x$? Hint, it is *not* $\cos x$). The idea in both cases is to switch into natural bases. Remember $x = e^{\ln x}$. So if we start with b^x we can write $b = e^{\ln b}$ and make good progress. $b^x = (e^{\ln b})^x$. Using this, (9) what is the derivative of b^x?

What about $\log_b(x)$? This we will resolve in a different fashion, similar to logarithmic differentiation. Let $\log_b(x) = y$, (10) solve for x. (11) Now take *natural* log of both sides, and solve for y. This should produce the familiar result $\log_b(x) = \frac{\ln x}{\ln b}$, but that denominator just a constant, so you should be able to (12) take the derivative from here.

(13) What are the integrals of b^x and $\log_b(x)$? [There is a catch to one of these that we will need to delay.]