Not a lot of wrap-up for Lab 24. We saw some polar graphs. They are nice. We will see some more here, and then finally, for our last worksheet, we will do some calculus with them. But, that's not where we are today. Because we don't have as much to do today, I'll stick this here: (1) Are there any questions about Lab 24? I would be very pleased to discuss any of it.

In lab we saw flowers, spirals and snails. Let's look again at the snails. Consider  $r = 2\cos\theta - 1$ . Start by making what I call an "auxiliary graph", i.e. a graph of  $y = 2\cos x - 1$ . This is a good time to remember your pre-calculus trigonometry graphing techniques. (2) Start with your graph of  $\cos x$ . (3) What effect does the 2 have? (4) What about -1?

Ok, so look at your graph, let's say from 0 to  $2\pi$ . Notice some points ... (5) when is it equal to 1? (6) -1? (7) -3? (8) 0? (takes a little more work, but is very important). More important than that is what happens in between. This allows us to fill up the entire graph. So, given the answers above, now that we have our auxiliary graph set up, if we start our polar graph at  $\theta = 0$ , (9) what is r? (10) What happens to r from  $\theta = 0$  to  $\theta = \pi/3$ ? So, draw your graph to show that. Make sure not to cross the spoke for  $\theta = \pi/3$ . (11) Now, from  $\theta = \pi/3$  to  $\theta = \pi$ , what happens to r? (Remember what it means for r to be negative). Great - we're half-way done. Now we just have to go back. (12) From  $\theta = \pi$  to  $\theta = 5\pi/3$  what happens to r? (again, don't cross the spoke). (13) Finally, what happens from  $\theta = 5\pi/3$  to  $\theta = 2\pi$ ? This process of working with an auxiliary graph is valuable and gives us good insights into what is happening and why. (14) Make sure you see how this matches with calculator produced graphs both in rectangular and polar coordinates.

Let's work to gather some of the graph information that we'll need for doing integration in §11.5. (15) In lab you had questions like "find a range of *theta* values corresponding to one petal of  $r = \sin 2\theta$ ." Make sure you can do that.

How about more of this kind? (16) Find the intersection points between r = 1 and  $r = 2\cos\theta$ . (17) Find the intersection points between  $r = \cos\theta$  and  $r = \sin\theta$ . One of the intersection points is clear visually and the other is clear from the trigonometry. Make sure you find both.

Here's a moderately interesting question ...(18) thinking about the snail from above, what is the intersection between  $r = 2\cos\theta - 1$  and r = 2? (19) Why might you think that they have no intersection? (20) Why do they?

I guess that's about what I have. We'll play more with calculus in our very last section.