371 Problem Set 67 Comments
Chapter 6

1. Not much to say. Either follow his hint, or you can invert and do a substitution more like a calculus $u$-substitution. This way you could start with the right-hand side and simplify using $u=\phi(t)$. In the end, this is a standard calc III result. If you look at your calc III book, wherever it is, whatever it is, you will find this proven. (Some be careful - Leibniz notation is not a fraction.) Also note $\gamma=(x, y)$, so it is inappropriate to claim $\eta=(x, y)$ also.
2. Use some formulas on the top of 105 . Then use formulas on the bottom of 85 for $d z$ and $d \bar{z}$. Remember the rules for wedge products - they are anticommutative. In the second case the sooner you can move away from $x$ and $y$ the easier.
3. presentation
4. On $\# 4$, there's nothing wrong with solving for $x$ and taking the positive square root (since we want the right-hand branch), but it's not as much fun as getting to use the hyperbolic trigonometry for its intended purpose. I also saw a creative answer with $x=\sqrt{3} \sec (t), y=\sqrt{3} \tan (t)$. I think that works out well, too.
5. The graph is on page 108. The parametric equations are much like those for the circle on page 50. They are $\gamma(t)=\left(\frac{3 t}{1+t^{3}} \cdot \frac{3 t^{2}}{1+t^{3}}\right)$. The reason we use $\frac{x d y-y d x}{2}$ is that $d \frac{x d y-y d x}{2}=d x d y$, and so integrating it around the boundary determines the area within. The area is $\frac{3}{2}$.
6. The same steps as in 5 . The answer is $\frac{2 k+1}{2}$.
7. Let $z=e^{1 \theta}$, then find $\bar{z}$ and $d \bar{z}$ The answer is 0 (remember $e^{2 \pi i}=1$ ). There's an interesting issue if $n=1$, but I wasn't going to push the issue.
8. You'll need total derivatives because $r e^{i \theta}$ depends on two variables. Then you need wedge products. There's a worry about whether we get $d r d \theta$ or $d \theta d r$. We're lucky because it is zero. I think $d r d \theta$ is probably correct. Again something interesting happens if $m=n$.
9. This is the final step for the Cauchy integral formula. You may not use it to prove. It's pretty easy to do. Pull $f(z)$ out of the integral because it doesn't depend on $\zeta$, then the integral is pretty simply $2 \pi i$. .
10. presentations
11. These questions are getting sillier. I hope this stops. 6.12 with a limit on the RHS is an equivalent definition of derivative, so for $g$ to be complex analytic it must a least be continuous which requires that $g(\zeta)=f^{\prime}(\zeta)$.
12. The two by definition of derivative are from calc I. The two which are power series are merely properties of power series. And the two by partial derivatives are the sum rule and product rule for derivatives. 6 steps, but shouldn't be too bad.
13. To prove the power series definition, differentiate term-by-term. I don't think it's subtle as long as we have our uniform convergence case. To prove the Cauchy Riemann equations we use that mixed partials are equal and change the order of differentiation, and there's a nice use of the series definition for $f$ to prove the limit definition for $f^{\prime}$. Remember $f^{\prime}=\frac{\partial f}{\partial z}$, not $\bar{z}$.
14. Discussed in class
15. Basically included in 15 ; strange to be asked twice.
16. Unattempted.

Chapter 7
7.1 The limits need to be 0 to $\pi$, not $2 \pi$.
7.2 Somewhat done in class.
7.3 Done in class.
7.4 Attempted in class - unresolved; generalisation requires reproving.
7.5 not attempted
7.6 done in attempted.
7.7 1-2) are about the same. They seem pretty direct, no real problems noticed with them. 3) is similar, but has poles of order $n$, and I think that's enough to make it different. It should be merely a matter of finding the residue at $a i$, by computing a $n-1$ st derivative - not terrible. or by finding the binomial expansion.
4) seems not too bad.
5) long but perseverance is rewarded. The answer is $(-1)^{j} 2 \pi\left(\frac{1}{2}\right)^{2 j+2 k}$. Requires binomial theorem and hunting for the residue as the -1 coefficient in the expansion.

6-7) not attempted. I am convinced that 7) is wrong in the book, and that the $\sin \left(\frac{\pi}{\alpha}\right)$ must not be in the denominator. I haven't finished the details, and I'm not sure that 6) is correct either - it might be, it might not be. I'm very interested in these problems. I'll give 2 extra points on the final to anyone who hands me either of these problems written completely and neatly before starting the exam.
8) Should have been popular.

9-10) not attempted.

