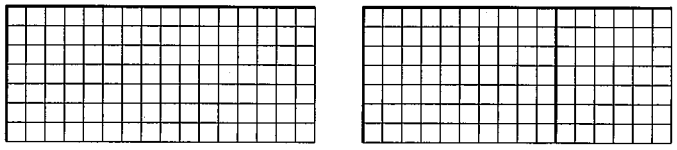
EXPLORATION 3.12 Understanding the Standard Multiplication Algorithm

When I taught fourth grade every day in 2003, I was surprised at how hard it was for the fourth-graders to master multidigit multiplication. Therefore, I went back and explored this process more. For the purpose of this exploration, assume that you do not have an algorithm for multiplying multidigit numbers.

1. The problem 16×7 is shown in two ways.

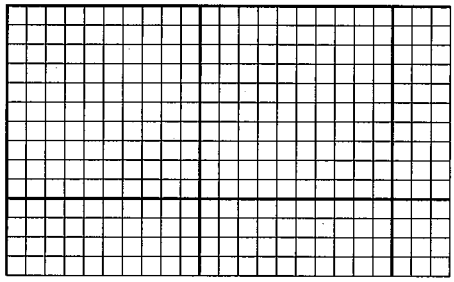


   a. How does that bold vertical line make it easier to find the answer to 16×7?
   b. Explain how you found the answer to 16×7.

2. Now using the Base 10 Graph Paper in the Appendix, cut out and find the answers to the following problems. Explain how you found the answer.

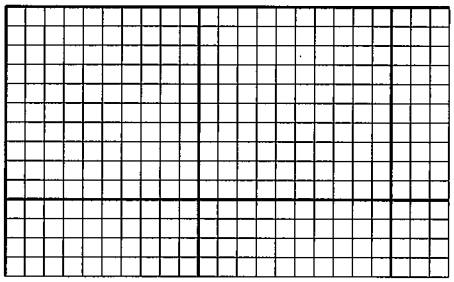
   a. $\begin{array}{r} 17 \\ \times\ 8 \\ \hline \end{array}$ b. $\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array}$ c. $\begin{array}{r} 23 \\ \times\ 7 \\ \hline \end{array}$ d. $\begin{array}{r} 27 \\ \times\ 8 \\ \hline \end{array}$

3. At the right are three 23×14 rectangles. One first step in finding a more efficient way to find the product is to use our base 10 knowledge to break this problem into 6 small rectangles (10×10, 10×10, 3×10, 4×10, 4×10, and 3×4). Write the six products in the diagram and add them.

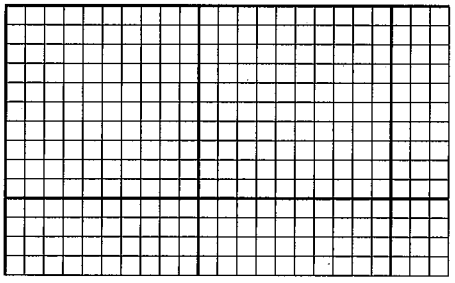


4. A next step is to decompose the problem into only two rectangles.

   a. Use a colored pencil or pen to draw a vertical line so that you have 20×14 and 3×14.
   b. Explain why the sum of these two problems is equal to 14×23.
   c. Multiply 14×23 longhand. What connections do you see between the partial products and what we just did?



5. a. Draw a horizontal line so that you have 23×10 and 23×4.
   b. Explain why the sum of these two problems is equal to 23×14.
   c. Multiply 23×14 longhand. What connections do you see between the partial products and what we just did?



6. Now combine the two cuts above to make four smaller problems out of 23×14. Write those four problems. Look at the expanded-form representation of 23×14, and look at your picture. Describe the connections you see.

$$\begin{array}{r} 20 + 3 \\ \times\quad 10 + 4 \\ \hline 80 + 12 \\ 200 + 30 \\ \hline \end{array}$$