

### EXPLORATION 5.13 Meanings of Operations with Fractions<sup>8</sup>

The purpose of this exploration is for you to develop your operation sense by connecting the problem with the operation.

#### PART 1: Matching problem situations with operations


For each of the seven problems below, do the following:

- a. Represent the problem with a diagram.
- b. Select the model that fits and briefly justify your choice:
  - + combine, increase
  - take-away, comparison, missing addend
  - × repeated addition, area, Cartesian product
  - ÷ partitioning, repeated subtraction, missing factor

If none of the models fits, then explain how you knew which operation was appropriate.

- c. Write a number sentence that would answer the question, *but do not determine the answer*.

As an example, one solution for Problem 1 is modeled below.

1. a. 
- b. "At first, I did the problem by adding. That is, how many  $\frac{3}{4}$  s can I add until I get to 12? But this didn't fit any of the models of the operations. Then I realized I could look at the diagram from "the other direction": How many  $\frac{3}{4}$  s could I take away until I ran out of medicine? Then I realized this was repeated subtraction, and the appropriate number sentence popped into my head."
- c.  $12 \div \frac{3}{4}$

#### Problems

1. A patient requires  $\frac{3}{4}$  of an ounce of medicine each day. If the bottle contains 12 ounces, how many days' supply does the patient have?
2. The label on a bottle of juice says that  $\frac{3}{4}$  of the bottle consists of apple juice,  $\frac{1}{6}$  of the bottle consists of cherry juice, and the rest is water. What fraction of the bottle is juice?
3. Freida had 12 inches of wire and cut pieces that were each  $\frac{3}{4}$  of an inch long. How many pieces does she have now?
4. Jake had 12 cookies and ate  $\frac{3}{4}$  of them. How many cookies did he eat?
5. Kareem had 12 gallons of ice cream in the freezer for his party. Last night Brad and Mary ate  $\frac{3}{4}$  of a gallon. How much ice cream is left?
6. The Bassarear family is driving from home to a friend's house, and Emily and Josh are restless. They ask, "How far do we have to go?" Their father replies that they

<sup>8</sup>This exploration is adapted from one developed by Ellen Davidson and Jim Hammerman at Education Development Center.

have gone 12 miles and that they are  $\frac{3}{4}$  of the way there. What is the distance from home to the friend's house?

7. Karla has  $\frac{3}{4}$  of an acre of land for her garden. She has divided this garden into 12 equal regions. What is the size of each region?

### **PART 2: Connecting whole-number and fraction contexts**

1. For each model that was not used in the seven problems in Part 1, write a story problem or explain why that model is not possible for fractions.
2. Some of the models (such as the combining context for addition) apply to fractions in the same way that they apply to whole numbers. However, some of the models do not work in the same way. Determine which models do not work in the same way and explain why. For example, let's say you decided that the area model for multiplication of fractions works only if the fractions are proper fractions; explain why.