

EXPLORATION 5.14 An Alternative Algorithm for Dividing Fractions⁹

The famous Indian mathematician Brahmagupta described an algorithm for dividing fractions that is actually easier to understand, in terms of *why* it works, than the traditional invert-and-multiply algorithm. The purpose of this exploration is to help you to understand this algorithm and why it works.

For each of the problems below, do the following:

- a. Represent the problem with a diagram and then solve the problem using the diagram.
- b. Describe what you did in order to arrive at your answer. It is important to consider carefully what you are actually doing, because careful thinking here increases the chances of the “breakthrough” in the next step.
- c. Try to connect what you did on paper to how the problem could be solved using only numbers. For example, in the first problem, the original number sentence is $4 \div 2/3$, but regardless of the diagram you draw, you will divide each of your four units into thirds, and thus you are now solving $12/3 \div 2/3$.

After completing the five problems, look for commonalities in all the problems that lead to a generalization (rule) that you could use in all the division problems. That is, you are trying to go from the concrete (pictorial) level to the abstract (numerical) level so that the manipulation of the numbers will make sense, just as you found for whole-number operations.

1. Josie’s Jammers have adopted a 4-mile stretch of highway to keep clean. Each afternoon they pick up trash. If they can clean $2/3$ of a mile per day, how many days will it take them to clean the whole section?
2. Chien has $2/3$ of a gallon of gasoline, and each time he mows the lawn, he uses $1/6$ of a gallon. How many times can he mow the lawn before buying more gasoline?
3. Lyra is building dog houses. She has $7\frac{1}{2}$ pounds of nails. Each dog house requires $1\frac{1}{4}$ pounds of nails. How many dog houses can she make?
4. Rita has $3\frac{3}{4}$ ounces of perfume and wants to sell the perfume in $3/8$ -ounce bottles. How many bottles of perfume can she make?
5. Jonah has $3\frac{3}{5}$ pounds of dog food. Each day his dog eats $2/5$ pounds. How many days’ worth of dog food does he have?

⁹This exploration has benefited from the influence of Ellen Davidson and Jim Hammesman at Education Development Center.