A key goal related to developing geometric understanding is to see relationships among shapes. Toward this end, in Tape 8 in the Annenberg Teaching Math K–4 video series, a second-grade teacher gives his students a square piece of paper that they fold three times. He then asks them how many different shapes they can make with this piece of paper. The only rule is that they can use only the folds that they have made—that is, no new folds. There is a lot of mathematics in this exploration. If you have access to this video, it is very interesting viewing.

   The exploration you will do is more sophisticated but similar.

1.   Take a blank sheet of paper that measures $8\frac{1}{2}$ by 11 inches. How can you determine how much to cut off—without using a ruler—so that you will have a square piece of paper? Do it.

2.   Now that you have a square sheet of paper, make the folds illustrated at the right.

3.   Determine how many different shapes you can make, using only the folds given. Several of the NCTM process standards are illustrated in this exploration.
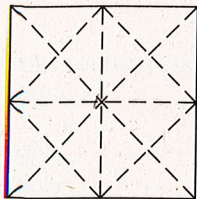
   First, one goal is to get as many shapes as possible (Problem Solving).

   Second, you will get more shapes if you have some kind of systematic approach, at some point, as opposed to just random folding (Reasoning).

   Third, you will get more shapes if you think outside the box; recall the 9-dots investigation in Chapter 1 (Reasoning).

   Fourth, you need to think about how you will display your results (Representation/Communication). If you simply sketch them roughly, you might miss some. For example, there are several squares that can be made that are different sizes. Also, when you compare your answers with other members of the class, some thought about how you want to represent your results will make it easier to find answers you have in common, and answers you have that your neighbor doesn't have, and vice versa.

   a. After you are finished, summarize your approach. This should be a short paragraph— enough detail so that a reader who only got a few shapes could understand your strategies and be able to get many more shapes, but not a blow-by-blow account of your thinking for each shape.

   b. Explain why you chose the representation system that you selected to display all the shapes you found.

4.   Summarize what you learned from this exploration.

   a. What mathematics did you learn?

   b. If you learned new ideas about problem solving, describe them.

   c. If you learned new ideas about reasoning, especially thinking systematically, describe them.

   d. If you learned new ideas about communication, describe them.

   e. If you learned new ideas about representation, describe them.

   f. If you see connections that you hadn't seen before, describe them.

   g. If you find that your attitudes and/or beliefs about geometry have changed, describe the change.