

Higher degree equations:

Gauß, Cauchy, Abel slide

Carl Friedrich Gauß 1777-1855 Brunswick (later to be Germany)

every $x^p - 1 = 0$ can be solved done using permutations of roots

regular 17-gon (at 19)

in 1801 which regular n-gons constructible (24)

fundamental theorem of algebra 1799 (22)

Disquisitiones Arithmeticae 1801 number theory

University of Göttingen

Augustin-Louis Cauchy 1789-1857 Paris, France

father government official for Bourbons

antirevolution, proroyalist

government service under Napoleon

1815 Cauchy clarifies some ideas of permutations.

Niels Henrik Abel (1802-1829) Norway

Norway part of Danish empire at 1800. Blockade of Norway by England – food for 7 children difficult to find. 1813 Sweden acquires Norway. After attempts for independence failed, oppression from Sweden.

Family in ministry and theology. Father involved in fight for independence. (alcoholic)

poor school teachers had left to staff university

new teacher (Holmboe) in 1817 (Abel 15)

by 16 studying work of Euler, Newton, d'Alembert, Lagrange, Laplace

1820 father died.

1821 enters new university at Christiania (later Oslo)

1822 begins work on solution of quintics by radicals / finishes university

Thinks has solved it. Submits. Finds error in review process.

works on elliptic integrals

seeks grant to travel to France / Germany

instead given money to learn languages for 2 years

1823 travels to Copenhagen – Christine Kemp fiancée

1824 (22) proved unsolvability of quintic in general

if it could be solved by radicals, there would be too many permutations (refers to work of Cauchy), which would lead to too many roots, instead of just 5.

wrote and published pamphlet himself to distribute. Sent to several mathematicians.

intro:

“The mathematicians have been very much absorbed with finding the general solution of algebraic equations, and several of them have tried to prove the impossibility of it. However, if I am not mistaken, they have not as yet succeeded. I therefore dare hope that the mathematicians will receive this memoir with good will, for its purpose is to fill this gap in the theory of algebraic equations.”

1825 travels to continent. Met Crelle there. leads to paper published in Crelle's journal in 1827

Holmboe appointed to only mathematics position in Norway (at Christiania).
was going to visit Gauß, but discouraged by negative reception of quintic paper.

“Not concerned with solution by radicals”

travels to paris – little positive reception.

remains in paris while waiting for answer on paper on elliptic integrals submitted to Cauchy – paper misplaced by Cauchy and ignored.

return to Berlin, 1826 – no money - sick

1827 in debt- returns to Norway

tutors children for \$, fiancée works as nanny

works on which equations can be solved by radicals

Abel gets more sick after traveling back in winter of 1828 Hemmoraging, tuberculosis

Crelle looks more for job for Abel

April 6, 1829 Abel dies

April 8, 1829 Crelle writes Abel with news of a appointment at Berlin Industrial Institute

Ideas of Galois

Evariste Galois (1811-1832) Paris area

kings slide

Time of modernization that grows out of post-revolutionary France. Metric system, statistics, medicine becomes scientific rather than mystical, in fact science becomes scientific. Beginning of social science (follows statistics). Birth of liberalism, radicalism, socialism, communism.

1800 Napoleon consolidates france, europe next

1803 11 December Berlioz: born La Cote Saint Andre father physician

1811 25 October Galois born, Bourg-la-Reine – father mayor

1813 Napoleon stopped in Russia

1814 Napoleon to Elba, Bourbons restored, Louis XVIII (“the Restoration”)

Galois' father leader of liberal republican movement

March 1815 100 days of Napoleon

Galois' father appointed as Mayor of Bourg-la-Reine

Vienna Congress – occupied by Austria, Russia, Prussia and Great Britain

Louis XVIII

White Terror

Father remains mayor nonetheless due to popular support

1816-Cauchy becomes member of French Academy after expulsion of republicans

also becomes professor at Ecole Polytechnique – research institution centred on mathematics

1818 Occupation of France Ends

1821 March Berlioz earns bachelor degree, follows father's wish to study medicine

1823 October Galois begins school at Lycée Louis-le-Grand, paris (educated by mother before)
longstanding reputation of academic excellence

strict regimented discipline in response to uprisings (teachers royalist, students republican) during 100 days
1824 excels in school, but homesick and bothered by politics
Louis XVIII dies
Charles X
royalist intolerance
Berlioz December 27 1824 - first rehearsal of first Mass, decides to become composer (disturbed by medicine)
Berlioz July 10 1825 - Mass performed in Paris, well received, strengthened resolve
Berlioz sometime 1824-Feb. 1826 at performance of of Sacchini's *Œdipe à Colonne*, encounter with enthusiastic mathematician. (pp. 56-57 Memoirs)

Berlioz & Mathematician, 1826?

The most curious scene of this sort that I remember took place one evening when *Œdipe* was being played. Though Sacchini held a far lower place in our esteem than Gluck, we nevertheless admired him greatly. I had dragged one of my friends, who cared for no art but that of billiards, but of whom I was nevertheless determined to make a musical proselyte, to the Opera. To my disgust, my friend was but moderately affected by the sufferings of Antigone and her father, so after the first act I gave him up as hopeless and went to a seat in front, where I should not be disturbed by his insensibility. As though to throw his impassibility into further relief, however, chance had placed on his right hand a spectator who was as impressionable as he was the reverse. I soon became aware of this. Derivis had made a great hit in his fine recitative. Absorbed though I was by the beauty and truly antique character of the scene, I could not help overhearing the dialogue which was going on behind, between my man, quietly peeling an orange, and his unknown neighbour, who was evidently suffering from the most intense emotion.

“Good God, sir, be calm.”

“No, it is too much; it is overwhelming! -- it is terrible!”

“But, sir, you are wrong to give way like this. You will be ill.”

“No; let me be ... Oh!”

“Sir, come sir, cheer up! After all *it is only a play*. Will you have a piece of this orange?”

“Ah! how sublime!”

“It is Maltese!”

“What heavenly music!”

“Do not refuse.”

“What music!”

“Yes; it is very pretty.”

During this discordant discourse we had got to the fine trio, *O doux moments*, which follows the scene of reconciliation; I was deeply affected by the penetrating sweetness of the simple melody, and hiding my face in my hands, was weeping silently. Scarcely was the trio ended when I felt myself raised from my seat by two powerful arms which clasped me like a vice; they belonged to the unknown enthusiast, who, perceiving that I alone of all his neighbours shared his feelings, and unable any longer to control his emotion, was embracing me with wild enthusiasm, exclaiming convulsively, “*Sac-r-r-re Dieu*, monsieur, how beautiful it is!” Without expressing any surprise, and turning my face all blurred with tears towards him, I said, “Sir, are

you a musician?" ... "No! but I could not love music more if I were." "What matter? shake hands; you are a right good fellow."

And thereupon, regardless of the jeers of the spectators who had formed a circle round us, and of the flabbergasted look of my orange-eating friend, we whispered to each other our names and professions. He was an engineer! a mathematician! In what strange soils can keep susceptibility thrive! His name was Le Tessier. I never saw him again.

Berlioz 1826 enters Paris Conservatoire begin formal study of music; cursed by mother, unsupported by father.

title slide again, for Galois at age 15

1826 Galois would be in senior class, was for a semester, and despite studying well
Held back in school, returned to junior class

Berlioz enters conservatoire competition but is disqualified by reason that piece was unplayable.
Berlioz criticises, citing the choice of a "mediocre pianist" to perform the work.

Berlioz 1827 meets Harriet Smithson, sees performing in Shakespeare as Ophelia and is smitten

1827 Begins mathematics – legend: Lagrange book Elements of Geometry intended for a two year course in two days – escape from other difficulties in world
neglects other subjects and classmates. Only mathematics
returns to senior class in fall. Teacher writes:

"He is under the spell of the excitement of mathematics. Therefore, in my opinion, his parents would be well advised to let him devote all his energies to this one subject. He is wasting his time here, and does nothing but irritate his teachers and force them to punish him continually."

1828 fails *Ecole Polytechnique* exam

returns to Lycée Louis-le-Grand, year with talented teacher Richard who realises Galois' abilities

discussions of current research of Legendre, Gauß, Lagrange and Cauchy

Berlioz 1828 enters composition competition for third time, is awarded second prize;
disappointment

Berlioz 1828 becomes a published critic

Berlioz 1829 begins *Symphonie Fantastique* composition

1829 April first paper published on roots of polynomials as continued fractions

April 6 – Abel dies (in part due to Cauchy's negligence in ignoring his paper for 3 years)

May – paper of work on equations (characterising which polynomial equations can and cannot be solved by radicals, inventing group theory [group, fields, field extensions, subgroup, normal subgroup] in the process) submitted to academy to be reviewed by Cauchy. Lost and ignored

July – father's suicide after being forced out of office by created scandal

fails EP exam again (answers too general, throws eraser)

1830 February begins at *Ecole Normale / Preparatoire* (Normal school) after barely entering much more religious and royalist than *Polytechnique*.

primitive Equations papers published in bulletin (how to solve solvable equations that are not accessible to Gauß's methods)

becomes more involved in politics.

France July 1830 revolution against bourbon Monarchy

France Charles X abdicates to Britain

France Louis Philippe proposed by Marquis de Lafayette as a compromise. Satisfied the upper middle class, but not the royalists, because placed by revolution and not the republicans, because still a king, but used flags and words of revolution, but acted by rules of 1814.

Berlioz on the July revolution:

“I hurriedly dashed off the last pages of my cantata to the tune of the dry thud of the stray bullets as they described a parabola over the roofs and struck close to my windows or on the walls of my room.

[afterward]

When peace was restored in Paris after a fashion, and Lafayette had presented Louis Philippe to the people as the best of all possible republics the Academie des Beaux-Arts resumed its functions.

Our cantatas were played, ... and as my piece (since burned) proved my conversion to the established faith, they at last, at last, at last, [B's 5th entry] awarded me the first prize.”

Galois Locked in school for July revolution, while EP students were leading revolution

Galois Cauchy leaves France with Bourbons

Galois Joins republican group “Friends of the People” – Galois enters 12/9, disbanded by king 12/31

Galois December 1830 - expelled from EN for his criticism of policies

Berlioz December 5 1830 - *Symphonie Fantastique* first performed in Paris

Berlioz and Harriet Smithson from SF liner notes.

Symphonie Fantastique written about Berlioz' passions for Smithson.

Berlioz' notes: A young musician of morbid sensibility and ardent imagination poisons himself with opium in a fit of amorous despair. The narcotic dose, too weak to result in death, plunges him into a heavy sleep accompanied by the strangest visions, during which his sensations, sentiments, and recollections are translated in his sick brain into musical thoughts and images.

5 movements:

- Reveries, Passions

He first recalls that uneasiness of soul ... those moments of causeless melancholy and joy, which he experienced before seeing her whom he loves; then the volcanic love with which she suddenly inspired him, his moments of delirious anguish, of jealous fury, his returns to loving tenderness and his religious consolations.

- A Ball

He sees his beloved at a ball.

- Scenes in the Country

Pastoral scenes inspire unusual calm. Then *she* appears once more, his heart stops beating, he is agitated with painful presentiments; if she were to betray him!

...One of the shepherds resumes his artless melody, the other no longer answers him. The sun sets ... the sound of distant thunder ... solitude...silence.

- March to the Gallows

He dreams that he has killed his beloved, that he is condemned to death and led to execution. The procession advances to the tones of a march which is now sombre and wild, now brilliant and solemn, in which the dull sound of the tread of heavy feet follows without transition upon the most resounding outburst. At the end, the *fixed idea* reappears for an instant, like a love-thought interrupted by the fatal stroke.

- Dream of a the Witches' Sabbath

He sees himself at the witches' Sabbath, in the midst of a frightful group of ghosts, magicians, and monsters of all sorts, who have come together for his obsequies. He hears strange noises, groans, ringing laughter, shrieks to which other shrieks seem to reply. The *beloved melody* again reappears, but it has lost its noble and timid character, it has become an ignoble, trivial, and grotesque dance tune; it is *she* who comes to the witches' Sabbath ... Howlings of joy at her arrival ... she takes part in the diabolic orgy ... Funeral knells, burlesque parody on the *Dies Irae*. Witches' dance. The Witches' dance and the *Dies Irae* together.

quote slide 1

1831 publishes letter criticising education

“Firstly, in science, opinions are unimportant. I want to know whether a particular individual is a good or bad teacher and I do not worry about his opinions on questions unrelated to his scientific field. It was therefore frustrating and scandalous to see how, during the Restoration, teaching posts were available to those who professed the most acceptable royalist and religious views. The situation has not improved. Mediocrity ... is rewarded.”

“Let us begin with high schools, from which pupils studying mathematics mostly aim to go on to the *Ecole Polytechnique*. What is done to assist them in this purpose? Is any attempt made to enable them to understand the true spirit of science, through explanation of the simplest methods? Is any effort made to make reasoning a second memory for them? Is there not, instead, some similarity with the way they learn French and Latin? For how much longer will these wretched youngsters be forced to do nothing but listen and repeat all day long? When will time be spared to reflect on this enormous mass of knowledge, so as to establish a cohesive pattern in these numerous isolated propositions, in these unrelated calculations? Truncated theories, overloaded with pointless observations, are taught in the minutest detail, while the simplest, most brilliant algebraic propositions are neglected. Actually ... great efforts are made to demonstrate ever longer, occasionally false calculations and arguments, and even obvious deductions.”

One cause of this situation: “Booksellers want thick tomes. The more there is in the books written by examiners, the more profit they will make from sales. This is why, every year, we see these huge new compilations, containing work taken from the great masters next to schoolboy essays.”

****Galois by brother slide****

Berlioz travels to Italy for 1831-1832 after winning prize. Is mostly despondent and displeased with Italy. Seduced by Camille Moke, who then marries another man. Berlioz plans to return to kill her and others, but is unexpectedly detained. Then suffers more in Italy.

math lessons and lectures

banquet held after the trial of those refusing to let National Guard be disbanded

May 9 "To Louis Philippe"

Prison 5/10-6/15 acquitted for threatening king (young & foolish?)

July 4 – paper rejected because Poisson, LaCroix did not understand

7/14 – Bastille celebration commemorating revolution

7/15 Arrested for wearing banned uniform of National Guard

until 4/29/1832

suicide attempt

returns to mathematics research, paper submitted from prison

****quotes slide 2****

"Firstly, the second page of this work is not crowded with surnames, first names, titles, rank and eulogies of some miserly prince, whose purse strings would be loosened when incense was being swung, only to be tightened up when the censer was empty. Neither is respectful homage to be seen, in letters three times the size of the title, to some high ranking scientist, whose protection is indispensable (I nearly wrote, inevitable) for any twenty year old aspiring writer. I do not acknowledge anybody's advice or encouragement as being responsible for the good qualities of my work. I would be a liar, if I did so. If I had a message for the great men of the world, or the great men of science, I swear it would not be one of thanks.

"It is the men of science who are responsible for the late appearance of the first of the two *memoires*, and the other category, for the fact that all this was written in prison. I must mention how my manuscripts got lots in the files of the fellows of the Institute of France, though I fail to understand such negligence by men who have Abel's death on their conscience'; not that I wish to be compared with that eminent geometrician.

"M. Poisson admitted that he had not understood it at all. This only proves to the eyes suffering from the self admiration of the author himself that M. Poisson did not want to, or could not understand but will undoubtedly prove to the eyes of the general public that my work means nothing.

****quote slide 3****

"All this leads me to believe that the work I am presenting to the public, will be received in the scientific world, with a smile of sympathy, that the most generous will attribute my failure to lack of ability. For a period, I shall be compared with Wronski, or those men who untiringly find a new solution to the squaring of a circle, every year.

"The examiners of candidates for admission to the *Ecole Polytechnique* will be irritated to learn that a young man they rejected twice has the presumption to write books that establish new ideas rather than mere textbooks.

"It is to be expected that frequent difficulties, I was unable to overcome, would present themselves in treating such new subjects, and venturing along such a new path. For this reason

the formula “I do not know” is often to be found in both these *memoires*. This is especially the case with the second one, which is the most recent. The readers I mentioned at the beginning will have reason to complain. Undoubtedly, the most precious book by the most learned author, is the one in which he includes everything he does not know. It is also undoubtedly true that the greatest harm an author can do his readers is when he disguises difficulties.”

1832 Cholera in Paris

18 March moved to clinic

meets Stephanie Dumotel, daughter of clinic owner/doctor

exit prison

Galois May 25 - letter to Chevalier disenchantment with life, broken love affair with Stephanie Dumotel. “How can I console myself after the source of the greatest happiness known to man came to an end in only one month? Happiness and hope are at an end, now surely consumed for the rest of my life. ... Pity never! Hatred, that’s all. Anyone who does not deeply feel this hatred for the present, does not really love the future.”

Charles X son returns to France – challenge to Louis Philippe?

time for revolution. Galois betrayed by Dumotel and mathematics, only remaining love is France. Stage a duel as a government conspiracy to kill him. Offer Galois as a Martyr for France.

night before notes of mathematics written.

May 31 dies ,paris

lamarque/tomb slide

At Funeral republicans decided to rally around death of General Maximilien Lamarque Napoleon’s appointed Marshall of France instead. Galois death in vain.

“June 5, 1832, Paris, France. General Jean Maximilien Lamarque is dead. The last major figure of Napoleon's Empire is gone. Seeing it as an obligation, the institutional monarch Louis-Phillipe holds a funeral procession for the Napoleonic war hero.

The student populace of Paris is in outrage. The last disciple of the Emperor is gone. As the procession moves down the Left Bank, students hand out arms and ammunition to the crowd, yelling "Republic or Death." The crowd roars in agreement. A street revolution begins.

Unlike the successful removal of Charles X two years earlier through a similar act, the King has the national guard on call and ready this time. By the end of the day, hundreds are dead. The State has won.

Berlioz December 9 1832 – Lelio first performed Paris “complement” to SF, redemption.

Berlioz October 3 1833 - marries Harriet Smithson

Berlioz March 8, 1869 dies

1870 Camille Jordan makes sense of Galois' material.
non-algebraic solutions involving elliptic modular functions discussed by Hermite [1858] and Klein [1884]. Parts of Galois writings still uninterpreted (sketchy details, esp from death memoir)

Conclusion: Galois was probably my first inspiration to become a mathematician, and as you can see I always feel strongly about him when discussing him. I modestly hope that I have inspired something in someone in this audience today. Thank you very much for this opportunity to share with you and thank you for your attention.