7.A2 Jordanus de Nemore on problems involving numbers

(a) *If a given number is separated into two parts whose difference is known, then each of the parts can be found.*

Since the lesser part and the difference equal the larger, the lesser with another equal to itself together with the difference make the given number. Subtracting therefore the difference from the total, what remains is twice the lesser. Halving this yields the smaller and, consequently, the greater part.

For example, separate 10 into two parts whose difference is 2. If that is subtracted from 10, 8 remains, whose half is 4. This is the smaller number and the other is 6.

(b) *If a given number is separated into two parts such that the product of the parts is known, then each of the parts can be found.*

Let the given number a be separated into x and y so that the product of x and y is given as b. Moreover, let the square of $x + y$ be e, and the quadruple of b be f. Subtract this from e to get g, which will then be the square of the difference of x and y. Take the square root of g and call it h. h is also the difference of x and y. Since h is known, then x and y can be found.

The mechanics of this is easily done thus. For example, separate 10 into two numbers whose product is 21. The quadruple of this is 84, which subtracted from the square of 10, namely from 100, yields 16. 4 is the root of this and also the difference of the two parts. Subtracting this from 10 to get 6, which halved yields 3, the lesser part; and the greater is 7.

(c) *If only one of two parts of a number is known, provided the sum of the product of the parts and the square of the unknown part is given, then the number can be found.*

Let the parts of the number be x and b, with b given. Also given is a, the sum of the product of the parts, and the square of x. Add z, equal to x, to $x + b$ so that the entire $x + b + z$ can be separated into $x + b$ and z. Now since $x + b$ times z equals the given a, and the difference of $x + b$ and z is the given b, then $x + b$ and z are found as are x and $x + b$.

For example, let 6 be one of the parts and 40 the sum of the product and the square. Double 40 and redouble to get 160. Add to this 36 to obtain 196 whose root is 14. From this subtract 6 and halve the remainder to yield 4. This is the unknown part that with 6 makes the desired number 10.

(d) *If the sum of the squares of the two parts of a given number together with the square of their difference is known, then both parts can be found.*

If the sum is subtracted from the square of the given number, what remains is twice the product of the two parts less the square of their difference. This becomes the sum of the squares of the parts less twice the square of their difference, and finally it is the given sum less thrice the square of the difference. When, therefore, the remainder is subtracted from the given sum, take one third of what is left. The root of this is the difference that was sought. Hence, all can be found.

For example, square the two parts of 10, and adding them to the square of their difference yields 56. Subtract this from 100 to get 44, which in turn is subtracted from 56. The remainder is 12, whose third is 4. The root of this is 2, the difference of the parts. Therefore the larger number is 6 and the smaller is 4.