se position

n Arabic is rendered into Latin as 'duarum falsarum posicionum regula' he rule of double false position'], by means of which the solutions of stions can be found. [...] Now, the two false positions are taken at will. at sometimes they are both smaller than is correct, sometimes both mes one larger and the other smaller: and the truth of the solutions is e proportion of the difference of one position from the other, that is what e rule of fourth proportion, where three numbers are involved; from (unknown) [number], that is the truth of the solution, is to be found; the f these is the difference of one number of false position from the other. the approximation, which becomes truth by means of this difference. The nainder, which is for approximating to the truth. We wish to show how he rule of weighing, so that by the demonstration of how these differences weighing you may be able to understand the subtle solution of other elchataieym.

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, $\frac{1}{4}$ and $\frac{1}{3}$ of which lie below ground; and are 21 *palmi*: we are asked for the ree: because $\frac{1}{4}$ and $\frac{1}{3}$ are found in 12, suppose the tree to be divided into 12 f which a third, and a quarter, that is 7 parts, are [i.e. make] 21 *palmi*: so roportion of 7 to 21, so will be [the proportion of] 12 parts to the length d because, when four numbers are proportional, the first multiplied by ual to the second multiplied by the third: so if you multiply the second of mentioned, 21, by the third 12, and divide by the first number mentioned, ey give 36 for the fourth (unknown) number, that is, for the length of the se 21 is three times 7, take three times 12, and you will similarly have 36. other method we use, namely that for the unknown thing you put any sen at will, which can be divided exactly into the fractions that are the question: and according to how the question is posed, with this nber you try to find the proportion that occurs in the solution of the example: the number we are asked to find in this question is the length of fore suppose it to be 12, since this can be divided exactly by 3, and by 4, en as divisors: and because it is said $\frac{1}{4}$ and $\frac{1}{3}$ of the tree are 21, take $\frac{1}{4}$ and $\frac{1}{3}$ supposed, they will be [i.e. will add up to] 7; and if this [sum] had 21 we should have arrived at the required answer, namely that the tree *almi*. But because 7 is not 21; it happens that as 7 is in proportion to 21, so tree will be to the one we seek, that is as 12 to 36: therefore we might say: I suppose, we obtain 7; what should I suppose so that we obtain 21? And essed this way, [we see that] we should multiply together the numbers at is 12 by 21; and the sum [*sic*, although he means product] should be e remaining number.

(*c*) *Birds problem*

Someone bought sparrows 3 for a penny (*denarius*), and turtle-doves 2 for a penny, and a dove 1 for 2 pence, and he had 30 birds of these kinds for 30 pence. We want to know how many birds he bought of each kind: I first supposed 30 sparrows for 10 pence, and kept back 20 pence, which make up the difference between 10 pence and 30; and I changed one of the sparrows into a turtle-dove, and the increase [in the money spent] brought about by that change was $\frac{1}{6}$ of a penny; because the sparrow was worth $\frac{1}{3}$ of a penny, and the turtle-dove was worth $\frac{1}{2}$ of a penny, that is $\frac{1}{6}$ of a penny more than the price of a sparrow: and, again, I changed one of the sparrows into a dove, and by that change improved my position by $1\frac{2}{3}$, that is by the difference there is between $\frac{1}{3}$ of a penny and 2 pence; and I made six of these $1\frac{2}{3}$s, and by making six of them obtained 10: and according to this I should change sparrows into turtle-doves and doves, until this change yielded the 20 pence which I kept back earlier: so I made six of them, and by so doing obtained 120; which I divided into two parts, one of which could be divided exactly by 10 and the other by 1; and the total of [the results of] the two divisions was not to be so large as 30; and the first part was 110, and the other 10: and I divided the first part, that is 110, by 10, and the second by 1, and I had 11 doves and 10 turtle-doves: taking these from the 30 kinds, there remained 9 for the number of the sparrows; which sparrows are worth 3 pence, and the 10 turtle-doves are worth 5 pence, and the 11 doves are worth 22 pence; and so from these three kinds of birds we shall have 30 for 30 pence, as was required.

And if we wish to have 29 birds for 29 pence, we may operate in the same way, that is we take the price of 29 sparrows, the cheapest birds, from the 29 pence, and the remaining [money] is taken six times, and thus gives 116; which we again divide into two parts, one of which is to be exactly divisible by 10, and the other by 1; and the sum of [the results of] the two divisions is not as large as 29; which parts can be constructed in two ways: firstly so that the first part is 110, and the second 6; and when 110 is divided by 10 we obtain 11 doves; and when 6 is divided by 1 we obtain 6 turtle-doves; taking these [i.e. their sum] from 29, there remains 12 for the number of the sparrows: or [i.e. secondly] we shall divide 116 into 100 and 16; and we shall divide 100 by 10, and 16 by 1, and we shall have 10 doves and 16 turtle-doves; and the remainder, to make the number up to 29, that is 3, will be sparrows; and thus we have solved this question in two ways.

And if we wish to have 15 birds for 15 pence I shall show this is not possible without a fractional number of birds. For example, if I were to subtract the price of 15 sparrows from 15 pence; and I take the remaining pence six times, which gives 60, this cannot be divided into two parts, one of which is to be divisible by 10 and the other by 1, so that from these two divisions [i.e. from the sum of the quotients] we obtain a number less than 15: for instance: if I divide 60 into 50 and 10; and divide 50 by 10, and 10 by 1, the results of the two divisions are 5 and 10; which together add up to 15, that is to the sum of all the birds; and thus there will be no sparrow in this purchase; because 5 doves are worth 10 pence, and 10 turtle-doves are worth 5 pence; and thus from these two kinds of birds alone we have 15 birds for 15 pence: and nor is there any other number less than 60 and more than 50 which can be divided exactly by 10; and a smaller number has no place here; for if we were to put 40 for one part there would remain 20 for the other part: from which [we have] that if 40 is divided by 10, and 20 by 1, there results from the sum of the quotients of the two divisions 24 birds: which has no place [here], since there must be 15 [birds]. But if we wished to consider fractions of birds, we would divide the abovementioned 60 into 55 and 5, and divide 55 by 10, giving us $5\frac{1}{2}$ doves: and divide 5 by 1, giving us 5 turtle-doves. So subtracting $5\frac{1}{2}$ doves as 5 turtle-doves from the 15 birds, there will remain $4\frac{1}{2}$ sparrows, whose price is 1 penny and a half; and the price of 5 turtle-doves is $2\frac{1}{2}$ pence; and the price of $5\frac{1}{2}$ doves is 11 pence; and thus from these three kinds of birds we have 15 birds for 15 pence.

(*d*) *The lion, the leopard and the bear*

A lion would eat one sheep in four hours; and a leopard [would eat it] in 5 hours; and a bear [would eat it] in 6: we are asked, if a single sheep were to be thrown to them, how many hours would they take to devour it? You will do this: for 4 hours, in which the lion eats a sheep, put $\frac{1}{4}$; and for the 5 hours the leopard takes put $\frac{1}{5}$; and for the 6 hours the bear takes, put $\frac{1}{6}$: and because $\frac{1}{6}$, $\frac{1}{5}$ and $\frac{1}{4}$ are found [exactly] in 60, suppose that in 60 hours they will devour the sheep. Then consider how many sheep a lion would eat in the 60 hours: since in four hours it devours one sheep, it is obvious that it would eat 15 sheep in the 60 hours; and the leopard would eat 12 as a fifth of 60 is 12. Similarly the bear would eat 10; since 10 is $\frac{1}{6}$ of 60. Therefore in 60 hours they [i.e. all three animals together] would eat 15 plus 12 plus 10 sheep, that is 37. So you will say: for the 60 hours, which I suppose, they will eat 37 sheep. What [time] should I suppose so that they will eat only one sheep? So multiply one by 60, and divide by 37, which gives $1\frac{23}{37}$. And in that number [of hours] they will have eaten up the sheep.