A Friendly Introduction to Mathematical Logic

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Errata

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1. Page 16. There is an extra $k+1$ in the last displayed equation.

2. Page 38. The statement of Exercise 4 is incorrect. Rather than letting σ be $(\forall x)(\exists y)\big[x < y \rightarrow x+1 = y\big]$, it should be $(\forall x)(\exists y)\big[x < y \rightarrow x+1 \neq y\big]$. The new σ can be made true or false in a one-element structure, depending upon the definition of $<^{\mathfrak{A}}$.

3. Page 83. Exercise 7. The parenthetical comment should read: (and is a consequence of the Soundness Theorem, Theorem 2.5.3).

4. Page 118. In the statement of Exercise 1, replace $\mathfrak{A} \subseteq \mathfrak{B}$ by $\mathfrak{B} \subseteq \mathfrak{A}$. (Corrected in the Second Printing.)

5. Page 185. There is a flaw in the exposition of the proof of the First Incompleteness Theorem in the case when AxiomOfA is not represented by a Δ-formula. In this case, the claim that $Deduction_A(\overline{c}, \overline{f})$ is a true Δ-sentence is false, and so Formula (5.2) does not follow.

   Rather, to establish (5.2) when AxiomOfA is not represented by a Δ-formula, we argue as follows: Since AxiomOfA is recursive, it is represented by a formula $AxiomOfA(e)$. Thus the formula $Deduction_A(c, f)$ clearly *defines* the set Deduction$_A$. The argument that $Deduction_A$ represents Deduction$_A$ is slightly tangential to the exposition of the text, but (in outline) it goes like this.

   Say that $\phi(x)$ is positively numeralwise determined if, for each $a \in \mathbb{N}$, $\mathfrak{N} \models \phi(a)$ implies $N \vdash \phi(\overline{a})$. Say $\phi(x)$ is numeralwise determined if both $\phi(x)$ and $\neg\phi(x)$ are positively numeralwise determined. Thus you can show that ϕ represents Q if and only if ϕ defines Q and ϕ is numeralwise determined. Therefore, to show $Deduction_A$ represents Deduction$_A$, we must only show that $Deduction_A$ is numeralwise determined.

   To do this, we prove an analog to Proposition 4.3.9, showing that atomic formulas are numeralwise determined, and that the collection of numeralwise determined formulas is closed under $\neg$, $\vee$, $\wedge$, and bounded quantification. Since $Deduction_A$ is in the smallest collection of formulas generated by these operations from the atomic formulas (together with $AxiomOfA$), it follows that $Deduction_A$ is numeralwise determined, and thus $Deduction_A$ represents Deduction$_A$.

   We need to establish formula (5.2) on page 185.

   Assume that $Thm_A(f)$ is true in the standard model. Then for some natural number c, the formula $Deduction_A(c, f)$ is also true in the standard model. Thus the ordered pair (c, f) is in the set Deduction$_A$.

   Since Deduction$_A$ is represented by the formula $Deduction_A$, this means that the set of axioms N proves the formula $Deduction_A(\overline{c}, \overline{f})$, and then N also proves the formula $Thm_A(\overline{f})$, as needed to establish (5.2).

6. Page 186. The statement and proof of Corollary 5.3.4 are incorrect. In the statement of the Corollary, an additional assumption is needed: Assume, in addition, that A is strong enough to prove all of the axioms of N.

   Then in the proof, rather than constructing θ as in Gödel I, we need to have θ be such that

   $$N \vdash [\theta \leftrightarrow \neg\gamma\left(\overline{\ulcorner\theta\urcorner}\right)].$$

   From the assumption that θ is provable-from-A, you can show that θ is false-in-$\mathfrak{N}$. In addition, from the assumption that θ is not provable-from-A, you can prove that θ is true-in-$\mathfrak{N}$.

   Thus θ is either true and unprovable or false and provable.

   Then the argument in Gödel I shows that the assumption that θ is false and provable leads to a contradiction, while the argument in 5.3.4 shows that θ can't be true and unprovable. Thus the assumption that Thm_A is recursive leads to a contradiction, as needed.

7. Page 192, line 4. Replace

$$A \vdash \mathit{Deduction}(\overline{a}, \overline{\ulcorner\rho\urcorner}).$$

with

$$A \vdash \mathit{Deduction}_A(\overline{a}, \overline{\ulcorner\rho\urcorner}).$$

(Corrected in the Second Printing.)

8. Page 193. Exercise 4. Rather than

$$N \vdash \left[\theta \leftrightarrow \neg \mathit{Thm}_A\left(\overline{\ulcorner\neg\theta\urcorner}\right)\right],$$

it should be

$$N \vdash \left[\theta \leftrightarrow \mathit{Thm}_A\left(\overline{\ulcorner\neg\theta\urcorner}\right)\right].$$

So we really do want θ to assert its own refutability. (Corrected in the Second Printing.)