Discrete Math

1. . Determine with function is a bijection from $\mathbb{R}^+$ to $\mathbb{R}$.

   (a) $f(x) = x(x-1)$.

   (b) $f(x) = \sqrt{x}$.

2. At least one of the functions in Problem 1 is one-to-one. Find one and use the FORMAL MATHEMATICAL definition (as was done in class) to prove your choice was correct.

3. At least one of the functions in Problem 1 is NOT onto. Find one and prove your choice was correct.

4. Graph $y = \lfloor x^2 + 1 \rfloor$.

5. How many bitstrings (binary strings) are there of length 7 or less have:

   (a) at least three 1s .

   (b) at least three 1s and two 0s (use inclusion exclusion)

6. How many positive integers between 0 and 999 are divisible by 6 or 12?

7. How many (ordered) strings of four distinct digits taken from {0,1,2,...9}

   (a) do not contain the same digit twice.

   (b) end with an even number and have no digit repeated.

   (c) end with an even number and have have exactly one digit repeated.

8. Show that among any 21 positive integers, three must have the same remainder upon division by 10.

9. How different bags of seven letters of the English are there if no bag can contain a repeated letter?

10. How many words of seven letters taken from the English alphabet can be formed?

11. For $A = \{a, b, c, d\}, B = \{2, 3, 5\}, C = \{c, e, f, g, h\}$. How many elements in $A \times B \times C$?

12. List the 3-sets of $\{1, 2, 3, 4, 5\}$? How many are should there be?

13. List the 2-sets of $\{1, 2, 3, 4, 5\}$? How many are should there be?

14. How many permutations of the letters A,B,C,D,E,F,G,H contain the string BD and EF, but not AG.

15. What is the coefficient of x^4y^7 in $(2x - 4y)^{11}$.

16. How many ways to give 43 pieces of candy to 6 children if:

    (a) there are no restrictions.

    (b) the two youngest children must get at least 3 pieces of candy.

17. Show by induction that $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{N}$.

18. Consider a collection of 27 cards where each card has either the number 0,1,2 AND the color red, green or blue, AND the shading light medium or dark. In other words, each card corresponds to an element in $\{0, 1, 2\} \times \{r, g, b\} \times \{L, M, D\}$. For example,(1,g,L) means the card has a 1 and green lightly shaded.

    Decide whether or not the following are equivalence relations on the $\{0, 1, 2\} \times \{r, g, b\} \times \{L, M, D\}$. If it is give the equivalence classes. If not, show (by example) which properties are not (refl, sym, trans.) satisfied:

    (a) x~y iff both are red

(b) x~y iff they are the same color

(c) x~y iff they are the same number

(d) x~y iff they are the same color or the same number

(e) x~y iff they agree at two attributes

(f) x~y iff they do not agree at any attributes

(g) x~y iff they have the same number and shading.

(h) x~y iff they either agree at each attribute or the disagree at each attribute.

(i) x~y iff they the number of x is < the number of y.

19. Draw $K_{2,5}$.

    (a) Does $K_{2,5}$ have an Euler circuit ? Give your reason(s). If "no", can you add some edges to $K_{2,5}$ to get a super graph (simple) that does have an Euler circuit? If yes, give such a picture of a supergraph.

    (b) Is $K_{2,5}$ planar. If yes, give a planar representation and indicate the number of regions formed.. If not show why.

20. Is there a simple graph with the degree sequence 3,3,4,4,4,5? Is so draw the graph.

21. Suppose there was a simple graph with 3,3,3,3,4,4,4,4,5,5,5,5,6,6,6,6. How many edges does it have?

22. Suppose $\begin{matrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{matrix}$ is the incidence matrix for a simple graph. Draw the graph and vertex color it with a mininumum number of colors.

23. Is the graph in problem 22 planar? Why or why not.