Goslo's perspective, traveling out

- The outbound trip takes $\Delta t_1 = \frac{L}{v} = \frac{10\,\text{ly}}{0.5c} = 20\,\text{y}$. Return will also take $\Delta t_2 = 20\,\text{y}$. (Neither of these is proper.)
- Goslo sends pulses every $T_s = 1\,\text{y}$. (That is proper.) He will be sending out 40 pulses total.
- When multiple messages are "in flight," they are spaced apart by a distance $d_1 = cT_s = 1\,\text{ly}$.
- Since the light has to catch up to Speedo, messages are received less frequently than sent. Call the receiving interval T_{r1}.
  - Starting at receipt of one message, the next message has to travel d_1 ***plus*** the extra distance that Speedo covers. Message travels $x_1 = d_1 + vT_{r1}$.
  - The time required for the light message to do that is $T_{r1} = \frac{x_1}{c} = \frac{d_1 + vT_{r1}}{c} = \frac{cT_s + vT_{r1}}{c} = T_s + \beta T_{r1}$, so $T_{r1} = \frac{T_s}{1-\beta} = 2T_s = 2\,\text{y}$.
- During outbound flight, Speedo receives $N_1 = \frac{\Delta t_1}{T_{r1}} = \frac{20\,\text{y}}{2\,\text{y}} = 10$ pulses.

When Speedo is at planet X, 10 more messages will be in flight.

Note that T_s is a proper time, but T_{r1} is not. Speedo measures the proper time of receipts.

Goslo's perspective, returning

- Return trip takes $\Delta t_2 = \Delta t_1 = 20\,\text{y}$. Messages are still sent out every $T_s = 1\,\text{y}$, and in-flight messages are still spaced apart by $d_2 = d_1 = 1\,\text{ly}$.
- Since Speedo is approaching, messages are received more frequently.
  - Starting at receipt of one message, the next message has to travel d_2 ***less*** the extra distance that Speedo covers. Message travels $x_2 = d_2 - vT_{r2}$.
  - So $T_{r2} = \frac{x_2}{c} = \frac{d_2 - vT_{r2}}{c} = \frac{cT_s - vT_{r2}}{c} = T_s - \beta T_{r2}$, and $T_{r2} = \frac{T_s}{1+\beta} = \frac{2}{3}T_s = \tfrac{2}{3}\,\text{y}$.
- On return flight, Speedo receives $N_2 = \frac{\Delta t_2}{T_{r2}} = \frac{20\,\text{y}}{\tfrac{2}{3}\,\text{y}} = 30$ pulses.

Great, so all pulses are received.

Speedo's perspective, traveling out

- The distance to travel is only $L' = \frac{1}{\gamma}L = \frac{10\,\text{ly}}{1.155} = 8.66\,\text{ly}$. So the outbound trip takes $\Delta t_1' = \frac{L'}{v} = \frac{8.66\,\text{ly}}{0.5c} = 17.32\,\text{y}$. (That's a proper time.)
- Goslo's clock is running slow, so he emits a pulse every $T_s' = \gamma T_s = 1.155\,\text{y}$
- Spacing of in-flight messages is extra big, as Earth recedes between sendings.
  - In the time between sending two messages, the first message travels cT_s' while the Earth travels vT_s' in the opposite direction.
  - So they are spaced apart a distance of $d_1' = cT_s' + vT_s' = (1+\beta)cT_s' = 1.732\,\text{ly}$
- Once in flight, messages simply approach Speedo at c, so the time between receipts is $T_{r1}' = \frac{d_1'}{c} = (1+\beta)T_s' = 1.732\,\text{y}$
- During outbound flight, Speedo receives this many pulses:

$$N_1 = \frac{\Delta t_1'}{T_{r1}'} = \frac{\left(L/\gamma v\right)}{(1+\beta)\gamma T_s} = \frac{L}{(1+\beta)\gamma^2 vT_s} = \frac{1-\beta^2}{1+\beta}\frac{L}{vT_s} = (1-\beta)\frac{L}{vT_s} = 0.5\frac{10\,\text{ly}}{0.5c(1\,\text{y})} = 10$$

So everyone agrees!
Note that d_1' has the same description as d_1, but **neither** is a proper distance.

Speedo's perspective, returning

This is a new, third frame, so I'll use double-primes. However, its speed is the same as the speed of the primed frame.

- Return trip takes $\Delta t_2'' = \Delta t_1' = 17.32\,\text{y}$, messages are sent every $T_s'' = T_s' = 1.155\,\text{y}$
- In-flight message spacing is extra small, as Earth approaches between sendings.
  - In the time between sending two messages, the first message and Earth travel the same distances (cT_s'' and vT_s''), but in the same direction.
  - So the messages are spaced apart a distance of $d_2'' = cT_s'' - vT_s'' = (1-\beta)cT_s'' = 0.577\,\text{ly}$.
- The time between receipts is $T_{r2}'' = \frac{d_2''}{c} = (1-\beta)T_s'' = 0.577\,\text{y}$.
- During return flight, Speedo receives this many pulses:

$$N_2 = \frac{\Delta t''}{T_{r2}''} = \frac{L/\gamma v}{(1-\beta)\gamma T_s} = \frac{1-\beta^2}{1-\beta}\frac{L}{vT_s} = (1+\beta)\frac{L}{vT_s} = 1.5\frac{10\,\text{ly}}{0.5c(1\,\text{yr})} = 30$$

So again, everyone agrees. Note that in order for the message spacing to change from d_1' to d_2'', some big changes have to happen during the turn around.