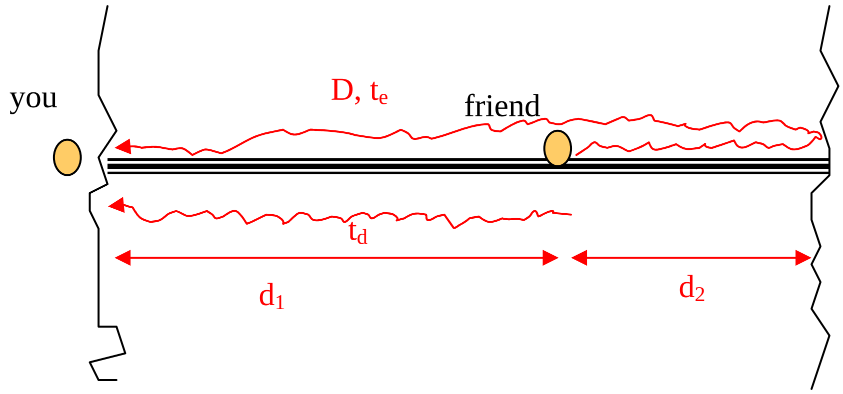
You and a friend are crossing a 100m long rope bridge across a gorge. You let him go first. When he is part way across, you hear him exclaim, “Uh-Oh!” Actually, you hear it twice, because of the echo off the far side. If the two sounds reach you 0.25s apart, how far across is your friend?



A Starting Comment

*You should **NOT** expect to “see” how you will solve this problem from the start. Like uncovering a fossil, expect to dig in with the tools you know, and gradually see how the pieces come together.*

First, draw the picture. That includes the sound paths, so that we can clearly see the lengths that we are working with.

Second define some variables and state some equations. Some of the variables have also been put on the picture, to make it easier to remember what they mean. (I’ve colored known variables blue.)

- s = speed of sound (in air) = 340m/s
- d_1 = distance from you to friend (what the problem asks for), and also the distance traveled by the direct sound
- t_d = time for direct sound travel
- d_2 = distance from friend to far side of gorge
- D = distance traveled by echoing sound
- t_e = time for echoing sound travel
- Δt = time between hearing the two sounds = 0.25s
- L = width of gorge = 100m

1: $s = \frac{d_1}{t_d}$

2: $D = d_2 + d_2 + d_1 = 2d_2 + d_1$

3: $s = \frac{D}{t_e}$

4: $\Delta t = t_e - t_d$

5: $L = d_1 + d_2$

At this point, we are in good shape, since there are 5 equations relating 5 unknowns.

Third step: eliminate the *undesired unknowns* by using substitution. Because d_1 is the ***desired*** *unknown*, we will NOT eliminate that by substitution.
I will use the following procedure (although this is not the only way to proceed): Start by combining two equations from above. Then at each step, take the previous step and incorporate one more of the equations above.

Solve #5 for d_2 and substitute into #2: $d_2 = L - d_1$

$$D = 2(L - d_1) + d_1 = 2L - 2d_1 + d_1 = 2L - d_1$$

Take that result for D and substitute into #3: $s = \frac{D}{t_e} = \frac{2L - d_1}{t_e}$

Take that result, solve for t_e and substitute into #4: $t_e = \frac{2L - d_1}{s}$ $\Delta t = \frac{2L - d_1}{s} - t_d$

Solve #1 for t_d and substitute into the previous equation: $t_d = \frac{d_1}{s}$ $\boxed{\Delta t = \frac{2L - d_1}{s} - \frac{d_1}{s}}$

A HA! After exactly 4 substitutions, getting rid of the 4 *undesired unknowns,* we are left with a equation containing only *knowns* and the *desired unknown.*
Now the fourth step, we need to solve this for d_1.

Make sure that d_1 is not in the bottom of any fractions — that is already true.
Get d_1 all by itself…

Multiply both sides by s: $s\Delta t = \left(\frac{2L - d_1}{s} - \frac{d_1}{s}\right)s = 2L - d_1 - d_1 = 2L - 2d_1$

Add the term with d_1 to both sides, to make it positive: $s\Delta t + 2d_1 = 2L$

Subtract the $s\Delta t$ term from both sides: $2d_1 = 2L - s\Delta t$

Divide by 2: $d_1 = L - \frac{1}{2}s\Delta t$

Hooray! Finally, the fifth step: substitute in the given quantities:

$$d_1 = 100\,\text{m} - \tfrac{1}{2}\left(340\frac{\text{m}}{\text{s}}\right)(0.25\,\text{s}) = 57.5\,\text{m}$$

A Final Comment

I have shown this in more detail than I would actually expect someone to do. I have done this to make very clear what I am doing along the way.
It might work for you to take shortcuts such as

- *don't define variables for known quantities, and put those numbers in the equations from the start.*
- *write down less detailed descriptions of the variables, especially it they are on the diagram.*
- *write much less text describing what you are doing at each step.*

Also, note that there are other ways to do this problem. You might use different variables, or see different initial equations. For instance, you might have noticed at the beginning that $d_1 + D = 2L$.