LAB 6: THE INVERSE SQUARE LAW

TOPIC OF INQUIRY

In this experiment we will examine the way that light energy becomes less intense to observers that are farther away. Specifically, if we assume that the light intensity is proportional to distance^n we want to determine the best value of n.

THEORY

I. Energy Conservation

Whether an incoherent light source is large like the sun or small like a light bulb, all its energy radiates uniformly directly outward away from it. This light energy is carried by photons. Each photon carries a small amount of energy that is inversely proportional to its wavelength. So, the power of a light source depends on the product of the energy per photon time the number of photons per second carrying that energy. Therefore, $P_L = k_1 R_p$ where P_L is the light power striking the detector, R_p is the rate of photon arrivals and k_1 is a constant of proportionality that depends on the wavelength of light.

If we enclose this source with a set of windows or screens, all its energy (and so, for any duration of time, all its photons and all its power) must pass through one of these windows. Consider such a window that spherically surrounds the light source. The surface area of the window is $4\pi r^2$. Since all the energy must pass through it, the intensity (power per area) of the light must decrease for larger spheres. Specifically, the optical intensity, I_{opt} will be $P_{source}/(4\pi r^2)$ where P_{source} is the total light power emitted by the source in all directions.. Every small portion of the spherical window will "see" this same light intensity. So, if a subset of the spherical window encloses 1 percent of the surface area of the total window, we expect that 1 percent of the total light power will be incident upon it. Therefore, if the area of the measurement window is constant, then the incident power will decrease with r^2. The fact that the surface area scales with r^2 is also the reason we see $1/r^2$ in both Newton's Law of Gravitation and in Coulomb's Law for electric force.

II. Solid Angles

One way to represent the area of the small subset of the spherical window is called the "solid angle". Its symbol is $\boldsymbol{\Omega}$, and it is defined as $\boldsymbol{\Omega}$ = (area of the sub-window)/$\boldsymbol{r^2}$. If the window is the entire sphere, then $\boldsymbol{\Omega = 4\pi}$. If the window is only 1 percent of the area of the full sphere, then the solid angle is $\mathbf{(0.01)4\pi\ =\ 0.126}$ The units of solid angle are steradians, or radians squared. However, since radians themselves are pure numbers, so are steradians. Writing the unit "steradian" is optional!

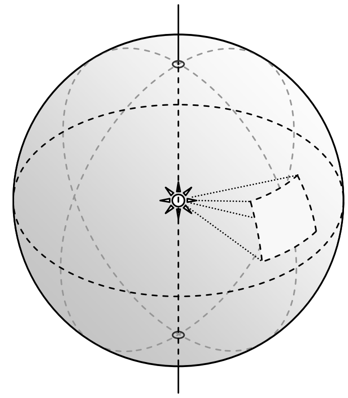


Figure 1: The area indicated here is only about 1 percent of the area of the sphere, or about 0.126 steradians.

III. Photovoltaic Cells

Once we start detecting light we have to be careful to distinguish between two quantities represented by the letter I, the Optical Intensity (I_{opt} discussed above) and the photocurrent signal generated in our detector (I_{pc} discussed below).

A photovoltaic "PV" cell is a (usually flat) semiconductor device that can create a small electric current that depends on the power of the light that shines on it. It is a quantum mechanical device that, if it was perfectly efficient, would create one electron's worth of "photocurrent" for each photon that strikes it, independent of how much energy it carried. A true PV cell is less than perfectly efficient. The photocurrent is $I_{pc}\ =\ k_2\ R_p$ where I_{pc} is the generated photocurrent, R_p is the rate of photons (photons per second) and k_2 is a proportionality constant that depends on the detector efficiency. Since the optical power also depends on R_p, we can combine equations to say $I_{pc} = \frac{k_2}{k_1}\ P_{opt}$ or more simply $I_{pc} = k\ P_{opt}$ where k is a constant that depends both on the wavelength of the light AND the efficiency of the detector. Since optical power is the product of optical intensity and collection area ($P_{opt} = I_{opt}A_{detector}$), more photocurrent is generated by the PV cell when either the light is more intense (more photons per second) or the cell itself is larger.

IV. Ohm's Law

It doesn't really make sense to say that the PV cell generates an electric current unless that current has somewhere to go, so a circuit is required. In this experiment, the electric current generated by the PV cell will flow from the PV cell, through a resistor, and back into the PV cell. One way to think of a resistor is that it is a device that converts electrical energy into thermal energy (heat). So, measuring the thermal power released by the resistor is the same as measuring the electrical power generated by the PV cell.

Recall that Ohm's law states that the voltage difference across the resistor will be proportional to both to its resistance and to the current through it which means $I_{pc} = \Delta V/R$. Since electrical power is $P_E = I\Delta V$, then for the resistor, $P_E = \Delta V^2/R$. So, measuring ΔV is a convenient way to determine electrical power!

APPARATUS

The equipment we will be using includes an incandescent lamp, a photovoltaic cell, a resistor, a ruler, and a voltmeter. The incandescent lamp will illuminate the PV cell, which will be used to measure light intensity. The resistor will be attached to the two output wires of the PV cell. The PV cell will face the light source, and at multiple distances from the source, the voltage across the resistor will be recorded.

EXPERIMENT

Start by measuring the area of the PV cell ($W \times H$) and the resistance R of your resistor, and record these values in your logbook. Also, measure the diameter of the light bulb, D_{bulb} , and of the aperture, D_a. Finally, measure r_0, the position of the aperture. Then, connect the two wires of the PV cell to the two ends of the resistor using the provided connectors. Finally, attach the two probes of the voltmeter to the two ends of the resistor using the provided connectors. Make sure that the voltmeter is in "Volts DC" mode, and that the output is in the millivolt range.

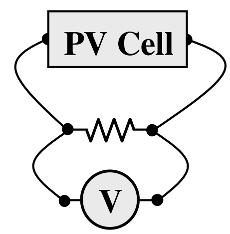


Figure 2: Wiring the equipment.

I. Determining the Power as a function of distance from the source

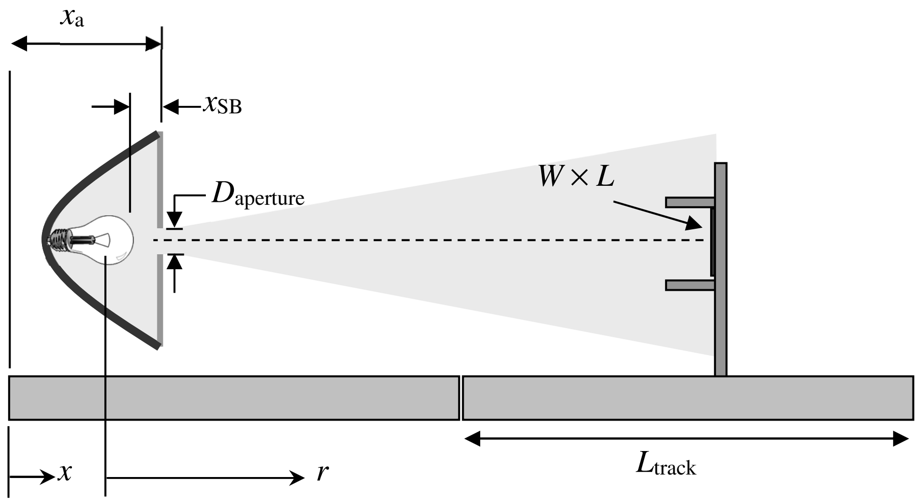
By itself, the light bulb is large compared to the size of the PV cell. Similarly, the size of the bulb may be large compared to the distance from the bulb to the cell. Each of these factors means that not all of the light striking the cell is radiating from exactly the same point. To minimize this issue, a small aperture is placed in front of the bulb. This aperture, rather than the entire bulb, will act as our light source. Because we are measuring light levels, you must ensure that you don't contaminate the experiment with light from your phones, laptop screens, etc. You will have to make your initial recordings on paper, relying on the ambient light in the room. Your results will be poor if the ambient light levels fluctuate during the experiment.

1) Even when the bulb is off, the PV cell will detect a small amount of ambient light from the room. Place the PV cell so that it is facing the source, about 1 m from the source. With the source still OFF, record the voltage difference V_0 across the resistor (in mV) when there is only ambient background light.

2) Turn the lamp on. Place the PV cell so that the front face of the cell is r = 10 cm from the face of the aperture. Make sure to use the shield around the PV cell to prevent extraneous light from reaching your detector. Record the voltage difference across the resistor. Repeat this measurement when the PV cell is x = 15 cm, 20 cm, …, 190 cm, and 195 cm from the aperture.

ANALYSIS

In Excel, make cells for the directly measured constants D_{bulb}, $D_{aperture}$, x_a, x_{sb} R, V_0, W, and H. Make a cell for x_0, the position of the filament (at the center of the bulb) that you calculate.



Make columns for x (position of the the cell) and ΔV (voltage across resistor in mV), and enter your measurements into excel. Make a new column for the distance from the filament to the cell ($r = x - x_0$) . Make a new column for the portion of the electric voltage that was generated by the lamp ($\Delta V_L = \Delta V - V_0$). Make a column for the photocurrent due to the lamp ($I_{pc} = \Delta V_L / R$).

Generally, we plot data so that it appears as a straight line, and we use the slope or intercept to determine a physical quantity. However, if photocurrent (or optical power) is proportional to r^n, and we hope to discover that $n \approx -2$, then it's not immediately clear that there is a straight line available to us. To extract n from our data, we'll use the following algebraic technique:

$$I_{pc} = ar^n$$

$$\ln I_{pc} = \ln(ar^n)$$

$$\ln I_{pc} = n \ln r + \ln a$$

Let's rename some of these quantities (conceptually, not literally). If $y = ln(I_{pc})$, $x = ln(r)$, $b = ln(a)$, and $m = n$, then this equation says that $y = mx + b$… a straight line! This manipulation suggests that we should make a new column for $ln(r)$, and a new column for $ln(I_{pc})$, and then plot them. The slope will be n, the exponent that determines the rate at which photocurrent declines with distance. We can then use "linest" in the usual way to determine both n and its uncertainty. If we wanted, we could also use the intercept to determine a, the coefficient of proportionality between photocurrent and distance squared.

LOGBOOK CALCULATIONS & QUESTIONS

Perform some calculations that are independent of our computation of n to determine the conversion efficiency of the PV cell. How much of the incident optical power on a cell gets converted to useable electrical power?

1) First, use the diameter of the aperture and the distance from the center of the light bulb (where the filament is located) to compute the solid angle of the aperture.

2) Knowing that the bulb is a 100 Watt bulb, how much power is emitted through the aperture?

3) Use the area of the PV cell to determine its solid angle when $r = 1$ m.

4) Compute the optical power that strikes the cell at this distance using the ratio of solid angles.

$$P_{cell} = P_{aperture} \frac{\Omega_{cell}}{\Omega_{aperture}}$$

5) Compute the electrical power dissipated in the resistor at this position, remembering that

$$P_E = \Delta V^2/R$$

6) Determine the efficiency of the PV cell. Efficiency is the ratio of output power electrical over input optical power, P_E/P_{cell}. If you multiply by 100, this will tell you the percentage of the incident light power that the PV cell converts into useful electrical energy.

7) The previous answer is breathtakingly low. In two or three sentences, explain what practical steps are necessary to use this kind of solar cell (not a more efficient one!) to generate useful green energy.