Applied Mechanics, Physics 313 Dr. Pogo
February 15, 2010

Exam #1

Name: SOLUTION

This exam is *closed notes and text*. You *may* use your calculators if you choose. Show all work for full credit. The value of each problem is indicated in [square brackets]. $\boxed{\text{Box}}$ your answers. Use any coördinate systems shown. All questions take place near the surface of the earth.

1. [15 points] A mechanic uses the pulley system shown to hold a 1200 pound engine. The diameter of the pulleys is much less than the length of cable between them, so you can assume that all the cable segments are vertical. Assume that the mass of each pulley and rope is small compared to the engine. Determine the tensions T_1 through T_9.

Note $T_4 = T_3 = T_5 = T_6 = T_7$

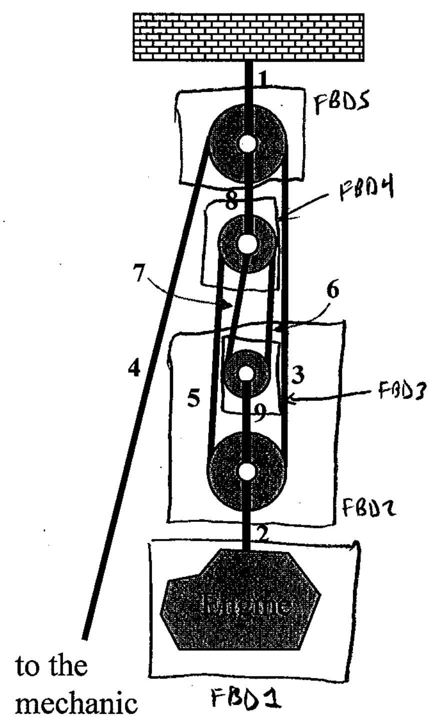
FBD2: $4T_4 = W$

$T_4 = \frac{W}{4} = 300\,lb$

FBD3: $2T_4 = T_9 = 600\,lb$

FBD4: $3T_4 = T_8 = 900\,lb$

FBD5: $T_1 = 2T_4 + T_8 =$



Answers:
$T_1 = 1500\,lb$
$T_2 = 1200\,lb$
$T_3 = 300\,lb$
$T_4 = 300\,lb$
$T_5 = 300\,lb$
$T_6 = 300\,lb$
$T_7 = 300\,lb$
$T_8 = 900\,lb$
$T_9 = 600\,lb$

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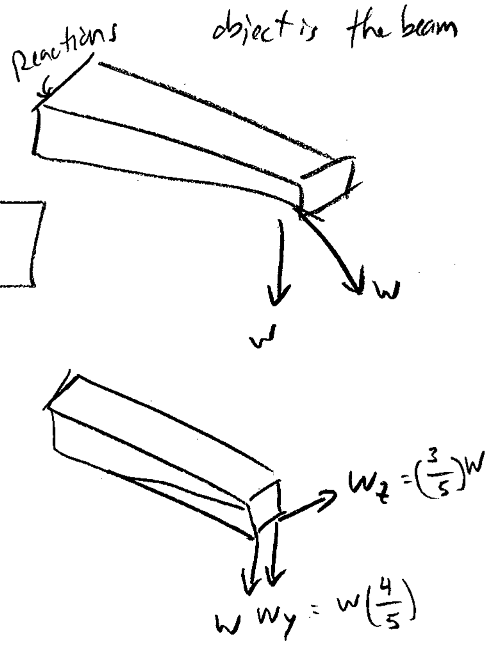
2. [30 points] A bucket of weight W is held by a rope, which passes over the *tiny* frictionless pulley at B, and is fixed to the ground at C. The connection of beam OA to the wall at O is weak; it will break if any of the reaction components there exceeds the following values:

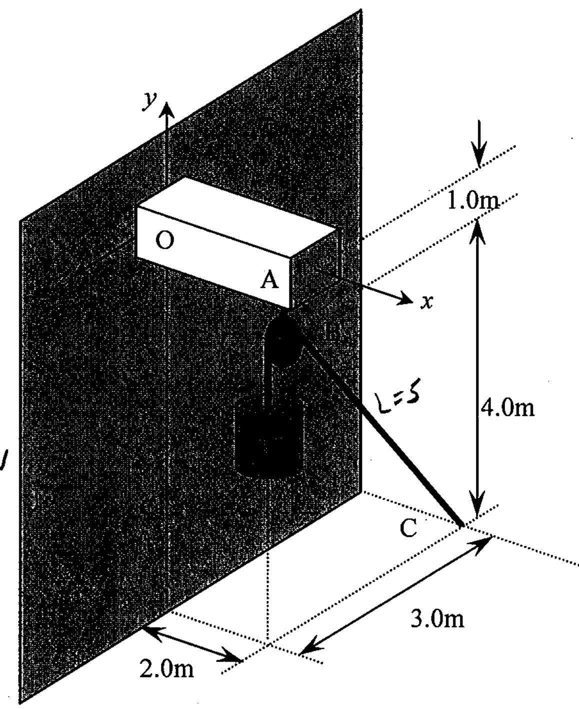
$|R_{Ox\text{-max}}| = 6$ kN $\quad |F_{Oy\text{-max}}| = 54$ kN $\quad |F_{Oz\text{-max}}| = 18$ kN

$|M_{Ox\text{-max}}| = 24$ kN·m $\quad |M_{Oy\text{-max}}| = 33$ kN·m $\quad |M_{Oz\text{-max}}| = 72$ kN·m

Probably, the connection will not break in all six ways at the same time.

a. [15] Find the actual six reactions as a *function* of W.
b. [10] Using the above restrictions, determine the actual maximum weight W.
c. [5] If the weight of part (b) is exceeded, which of the six restrictions are violated?





$\boxed{|F_{Ox}| = 0}$

$\boxed{|F_{Oy}| = W + W_y = \frac{9}{5}W = 1.8W}$

$\boxed{|F_{Oz}| = \frac{3}{5}W = 0.6W}$

$|M_{Ox}| = (1\text{m})\,W_z$

$\boxed{|M_{Ox}| = (0.6W)(1\text{m})}$

$|M_{Oy}| = (W_z)(2\text{m})$

$\boxed{|M_{Oy}| = (1.2W)(1\text{m})}$

$\boxed{|M_{Oz}| = (1.8W)(2\text{m}) = (3.6W)(1\text{m})}$

Using F_{Ox}; $W \to \infty$

F_{Oy}; $W = 30$ kN

F_{Oz}; $W = 30$ kN

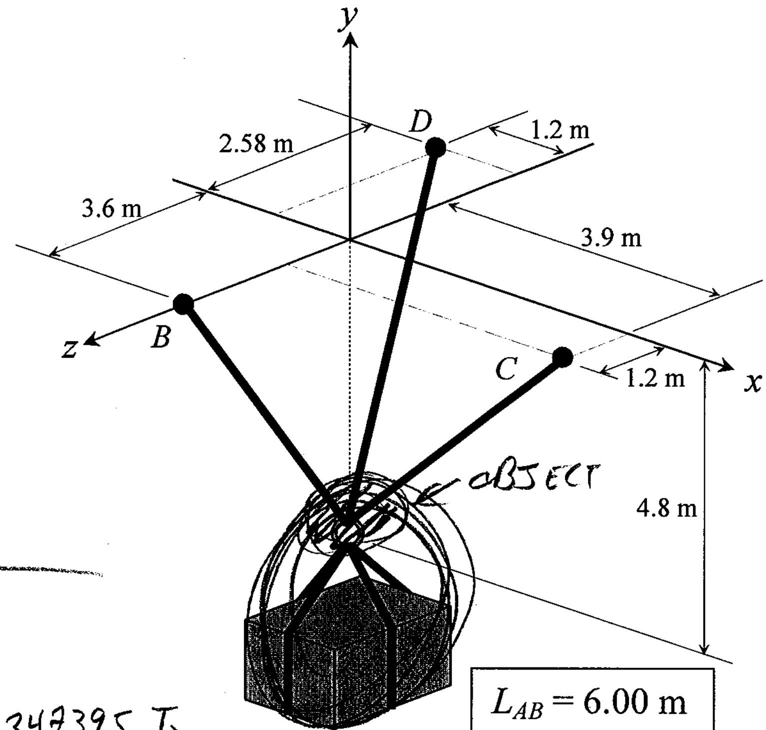
M_{Ox}; $W = 40$ kN

M_{Oy}; $W = 27.5$ kN

$\boxed{M_{Oz};\ W = 20 \text{ kN}}$

Restriction : $W_{MAX} = 20$ kN
Failure because of M_z.

3. [25] The crate shown hangs from the ceiling by three ropes, and has a weight W = 3600 N.
   a. [8] Find T_{AB}.
   b. [8] Find T_{AC}.
   c. [7] Find T_{AD}.



$L_{AB} = 6.00$ m
$L_{AC} = 6.30$ m
$L_{AD} = 5.58$ m
$W = 3600$ N

$\Sigma F_y = 0$

$T_{By} + T_{Cy} + T_{Dy} = W$

$T_B \frac{L_y}{L_B} + T_C \frac{L_y}{L_C} + T_D \frac{L_y}{L_D} = W$

$\frac{T_B}{6} + \frac{T_C}{6.3} + \frac{T_D}{5.58} = \frac{W}{4.8}$ [1]

$\Sigma F_x = 0 \quad T_{Cx} = T_{Dx}$

$(T_C)\left(\frac{3.9}{6.3}\right) = T_D\left(\frac{1.2}{5.58}\right) \rightarrow T_C = 0.347395\, T_D$ [2]

$\Sigma F_z = 0 = T_{Bz} + T_{Cz} - T_{Dz}$

$(T_B)\left(\frac{3.6}{6}\right) + T_C\left(\frac{1.2}{6.3}\right) = T_D\left(\frac{2.58}{5.58}\right)$ [3]

USING [2], [3] $\rightarrow (T_B)(0.6) = T_D(0.4623656 - 0.06617)$

$T_B = T_D(0.66033)$ [4]

Using [1], [2], [4] $\rightarrow$ $(T_D)(0.110055) + (T_D)(0.05514) + (0.17921)T_D = 750$ N

$\boxed{T_D = 2193 \text{ N}}$

[2] $\rightarrow$ $\boxed{T_C = 762 \text{ N}}$

[4] $\rightarrow$ $\boxed{T_B = 1417 \text{ N}}$

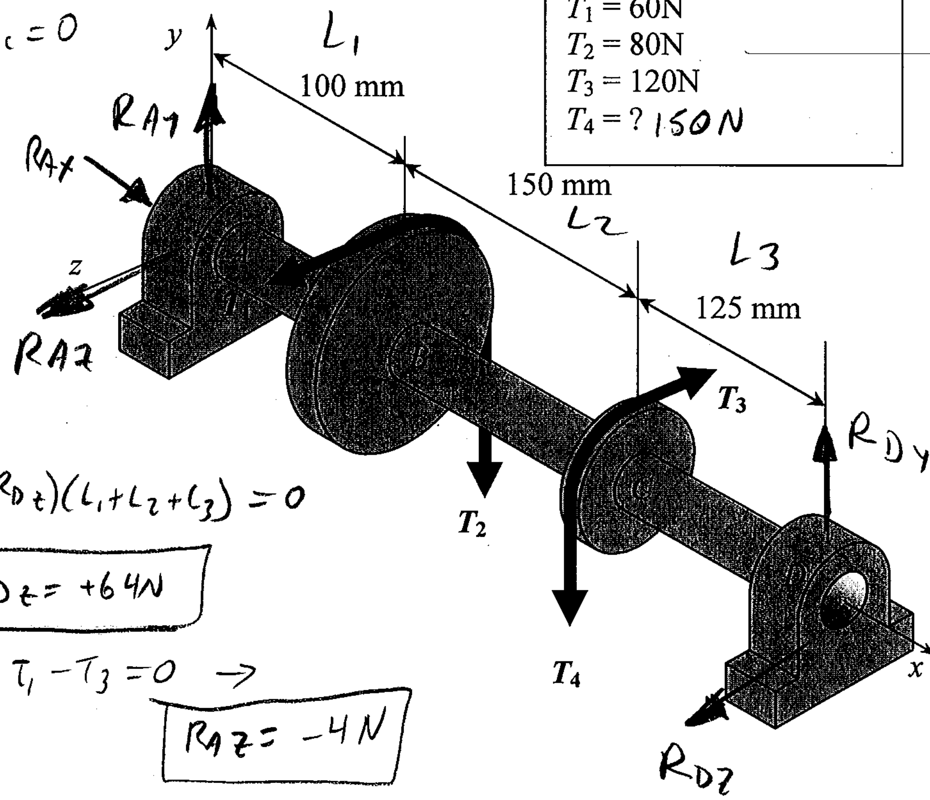
4. [35 points] Belts pass *with friction* over wheels *B* and *C*, which are welded to shaft *AD*, as shown. The shaft rotates at constant speed. Neglect the weight of the shaft and wheels.
   [5] a. Determine the tension T_4.
   b. Determine the reactions exerted on the shaft by the bearings at *A* and *D*. Note that the bearing at A *can* exert an axial force on the shaft, but the bearing at D cannot.

Radius B = r_B = 75 mm
Radius C = r_C = 50 mm
T_1 = 60N
T_2 = 80N
T_3 = 120N
T_4 = ? 150 N

I. $\Sigma M_x = 0$

$$+T_1 R_B - T_2 R_B + T_4 R_C - T_3 R_C = 0$$

$$\boxed{T_4 = 150\,N}$$



II. $\Sigma F_x = 0 \rightarrow \boxed{R_{Ax} = 0}$

III. $\Sigma M_{Ay} = 0$

$$(-T_1)(L_1) + (T_3)(L_1 + L_2) - (R_{Dz})(L_1 + L_2 + L_3) = 0$$

$$\boxed{R_{Dz} = +64\,N}$$

IV. $\Sigma F_z = 0 \rightarrow R_{Az} + R_{Dz} + T_1 - T_3 = 0 \rightarrow$

$$\boxed{R_{Az} = -4\,N}$$

V. $\Sigma M_{Az} = 0$

$$-T_2 \cdot L_1 - T_4(L_1 + L_2) + R_{Dy}(L_1 + L_2 + L_3) = 0$$

$$\boxed{R_{Dy} = 121.33\,N}$$

VI. $\Sigma F_y = 0 \rightarrow R_{Ay} + R_{Dy} - T_2 - T_4 = 0$

$$\boxed{R_{Ay} = 108.66\,N}$$