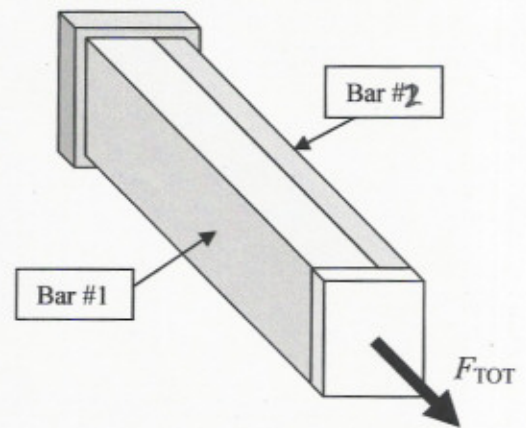


Determine:

- I. As  $F_{TOT}$  is increased, when is plasticity reached?
- II. What is the maximum possible  $F_{TOT}$ ?
- III. Find  $\delta$ ,  $\sigma_1$ , and  $\sigma_2$  if  $F_{TOT} = 189$  kips.
- IV. Find  $\delta_p$ ,  $\sigma_1$ , and  $\sigma_2$  if  $F_{TOT}$  is then reduced to zero.



Given:  $A_1 = 3 \text{ in}^2$   
 $E_1 = 15 \times 10^6 \text{ psi}$   
 $\sigma_{y1} = 45 \times 10^3 \text{ psi}$   
 $A_2 = 2 \text{ in}^2$   
 $E_2 = 30 \times 10^6 \text{ psi}$   
 $\sigma_{y2} = 36 \times 10^3 \text{ psi}$   
 $L = 40 \text{ in}$

I. Both pieces have the same  $\delta$ ...

$$\delta_{y1} = \frac{F_{y1}L}{A_1 E_1} = \frac{\sigma_{y1}L}{E_1} = 0.12 \text{ in at yield}$$

$$\delta_{y2} = \frac{\sigma_{y2}L}{E_2} = 0.048 \text{ at yield} \rightarrow \text{Since } \delta_{y2} < \delta_{y1},$$

Bar 2 will yield first.

At this time,  $\delta_1 = \delta_2 = 0.048 = \frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2}$

So  $F_1 = 54$  kips, and  $F_2 = 72$  kips

$F_{TOT} = F_1 + F_2 \rightarrow F_{TOT, yield} = 126$  kips

II.  $F_{1, max} = \sigma_{y1} A_1 = 135$  kips, and  $F_{2, max} = 72$  kips  $\rightarrow F_{TOT, max} = 207$  kips

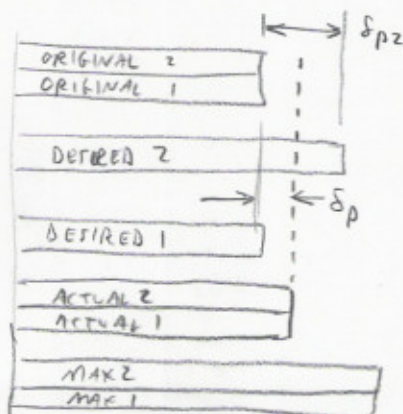
III. Bar 1 is still elastic.  $F_{TOT} = F_1 + F_2$ ; but  $F_2 = F_{2, max} = 72$  kips  $\rightarrow F_1 = 117$  kips

$\sigma_1 = \frac{F_1}{A_1} = \sigma_1 = 39$  ksi

$\sigma_2 = \frac{F_2}{A_2} = \sigma_{y2} = \sigma_2 = 36$  ksi

Using bar 1:  $\delta = \frac{F_1 L}{A_1 E_1} = \delta = 0.104$  in

IV. If the bars were not connected,  $\delta_{p2} = \delta_{max} - \delta_{yield2} = 0.104 \text{ in} - 0.048 \text{ in} = 0.056 \text{ in}$  (desired)  
 But unfortunately, they are connected! So, they exert internal forces on each other.



$F_1 = -F_2$   
 $\delta_p = \frac{F_1 L}{A_1 E_1}$ , and  $\delta_p - \delta_{p2} = \frac{F_2 L}{A_2 E_2}$  (expected to be negative)

$\frac{F_1 L}{A_1 E_1} - \delta_{p2} = \frac{-F_1 L}{A_2 E_2} \rightarrow$

$\delta_{p2} = F_1 L \left( \frac{1}{A_1 E_1} + \frac{1}{A_2 E_2} \right)$

$F_1 = 36$  kips  $\sigma_1 = +12$  ksi  
 $F_2 = -36$  kips  $\sigma_2 = -18$  ksi  
 $\delta_p = +0.032$  in