Basic Frictionless Compressible Nozzle Information (using air)

Air: $c_p = 1005$ J/kgK $c_v = 718$ J/kgK $R = 287$ J/kgK $k = 1.40$

Properties that are constant everywhere: T_0, $\dot{m}$

Properties that are separately constant on each side of a shock: p_0, a_0, ρ_0, A^*, p^*, a^*, ρ^*, s.

Subscript 0 indicates "stagnation" properties (where $v = 0$), and notation "*" means "sonic" properties (i.e., where $v = a$).

Properties that vary everywhere are p, a, ρ, A, T, v, $Ma = v/a$.

Flow is driven through the nozzle by some pressure gradient (e.g., given $p_{0\text{inlet}}$ and $p_{0\text{outlet}} = p_{\text{atm}}$):

Will the flow ever become "sonic"? (i.e., will it reach Mach 1): Only if $p^*_{\text{inlet side}} \geq p_{\text{0outlet side}}$

If it becomes "sonic", it does so at the minimum cross-sectional area A^*, called the "throat". The flow is subsonic prior to the throat. If it is supersonic, it is supersonic after the throat. If it is supersonic anywhere, there will eventually be a standing shock that reduces the flow to subsonic again eventually.

Functions valid everywhere on either side of a standing shock:

$$p = \rho R T \qquad a = \sqrt{kRT}$$

$$T^* = T_0\left(\frac{5}{6}\right) \qquad p^* = p_0\left(\frac{5}{6}\right)^{3.5} \qquad \rho^* = \rho_0\left(\frac{5}{6}\right)^{2.5} \qquad a^* = a_0\left(\frac{5}{6}\right)^{0.5}$$

$$T_0 = T\left(1+0.2Ma^2\right) \qquad p_0 = p\left(\frac{T_0}{T}\right)^{3.5} \qquad \rho_0 = \rho\left(\frac{T_0}{T}\right)^{2.5} \qquad a_0 = a\left(\frac{T_0}{T}\right)^{0.5}$$

Ma as a function of A: $$\frac{1}{Ma}\left(\frac{\left(1+0.2Ma^2\right)^3}{1.728}\right) - \frac{A}{A^*} = 0$$ (must know A^*, too)

$$A^*_{\min} = \frac{\dot{m}_{\max}\sqrt{RT_0}}{0.6847315\,p_0} \qquad \dot{m} = \rho A v \qquad v_{\lim} = \sqrt{2c_pT_0}$$

Property changes immediately across the shock (1 → 2):

$$Ma_2^2 = \frac{2+0.4Ma_1^2}{2.8Ma_1^2-0.4} \qquad \frac{p_2}{p_1} = \frac{2.8Ma_1^2-0.4}{2.4} \qquad \frac{\rho_2}{\rho_1} = \frac{2.4Ma_1^2}{2+0.4Ma_1^2}$$

$$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \left(\frac{2.4Ma_1^2}{2+0.4Ma_1^2}\right)^{3.5}\cdot\left(\frac{2.4}{2.8Ma_1^2-0.4}\right)^{2.5} \qquad \frac{A_2^*}{A_1^*} = \frac{Ma_2}{Ma_1}\cdot\left(\frac{2+0.4Ma_1^2}{2+0.4Ma_2^2}\right)^3$$