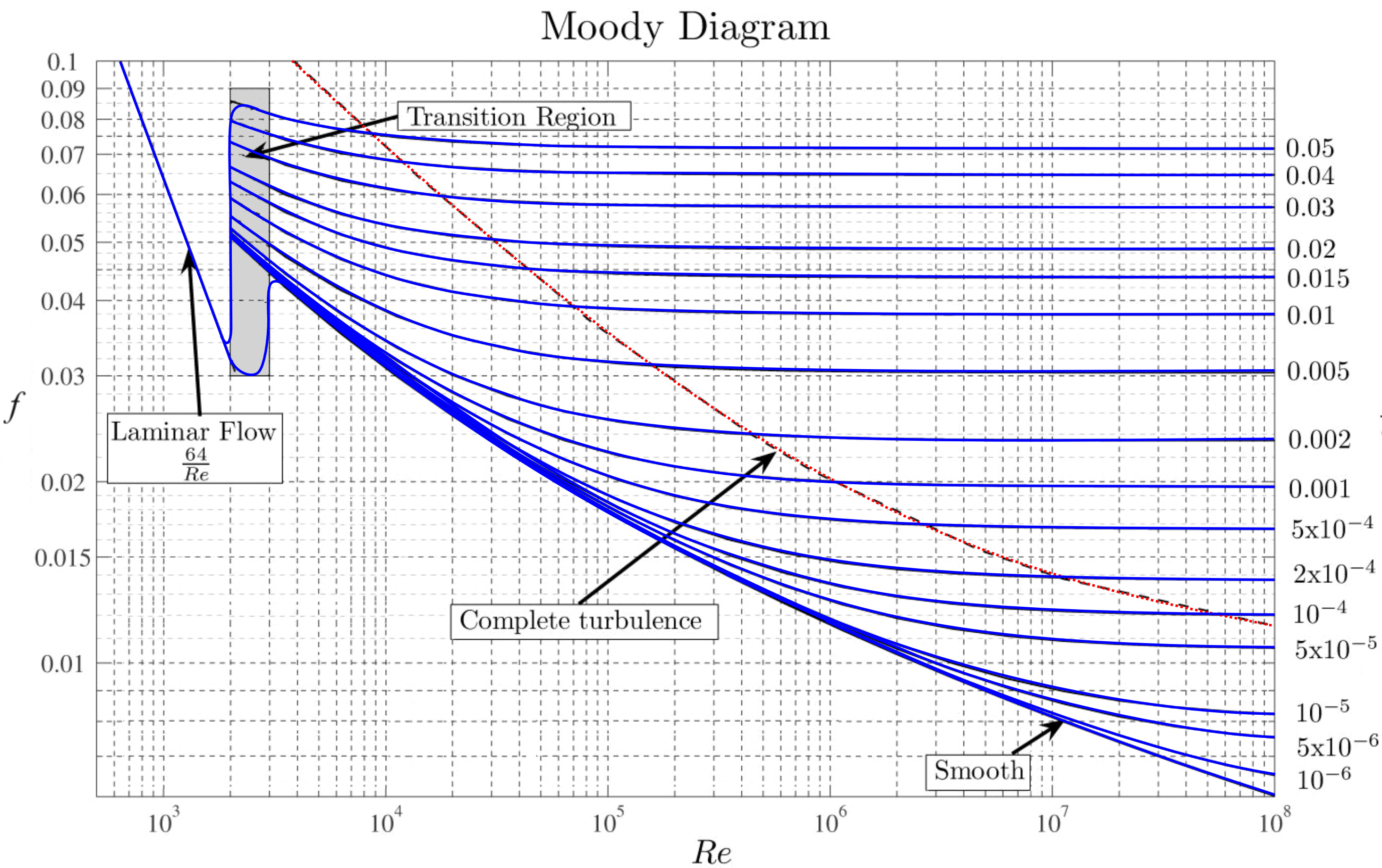
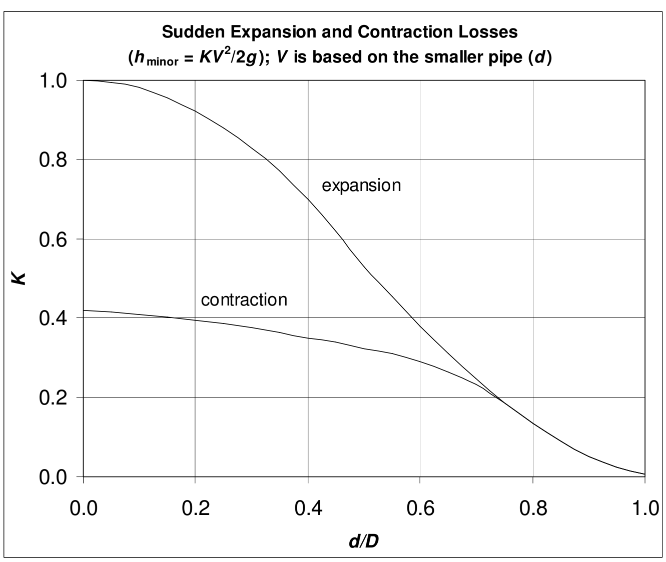
Pipes





Entrance loss coefficient: $K = 0.5$ (sharp edge)
Exit loss coefficient: $K = 1.0$ (any edge shape)

Roughness Values for Ducts:

Material	**Condition**	***e* (mm)**
Steel	Sheet metal, new	0.05
	Stainless, new	0.002
	Commercial, new	0.046
	rusted	2.0
Iron	Cast, new	0.26
	Wrought, new	0.046
	Galvanized, new	0.15
Brass	Drawn, new	0.002
Plastic	Drawn tubing	0.0015
Glass	-	Smooth
Concrete	Smoothed	0.04
	Rough	2.0
Rubber	Smoothed	0.01

Basics

$g = 9.8\ \mathrm{m/s^2}$

$\rho_{\mathrm{water}} = 998\ \mathrm{kg/m^3}$

$P_{\mathrm{atm}} = 101.3\ \mathrm{kPa}$

$\boldsymbol{V} = u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$

$\tau = \mu\, du/dy$

$\nu = \mu/\rho$

$\mathrm{SG} = \rho/\rho_{\mathrm{H2O}}$

Ideal gas: $P = \rho R_{\mathrm{gas}} T$

$\vec{F} = -\int p\, d\vec{A}$

$P_{\mathrm{gage}} = P_{\mathrm{abs}} - P_{\mathrm{atm}}$

$\bar{x} F_{total} = \sum x_i F_i = \int x\, dF$

Hydrostatics

$\vec{\nabla} p = \rho \vec{g}$

$p_2 - p_1 = \rho g (z_1 - z_2)$

$F_{\mathrm{buoyant}} = +\rho_w g V_{\mathrm{submerged}}$

$W = +\rho_o g V_{\mathrm{TOT}}$

$\rho_o/\rho_f = V_{\mathrm{submerged}}/V_o$

$-\nabla p + \rho(\boldsymbol{g} - \boldsymbol{a}) = 0$

$\boldsymbol{g}_{\mathrm{eff}} = \boldsymbol{g} - \boldsymbol{a}$

Boundary Layers

$\dfrac{\delta}{x} \approx 5.0\left(\dfrac{\upsilon}{Ux}\right)^{0.5} = \dfrac{5.0}{\mathrm{Re}_x^{0.5}}$ (Blasius: laminar)

$\dfrac{\delta}{x} \approx \dfrac{0.16}{\mathrm{Re}_x^{1/7}}$ (turbulent)

Pipe Flow

$$\left(\frac{P_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) - \left(\frac{P_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) = \frac{(u_2 - u_1)}{\mathrm{g}} - \frac{1}{g}\frac{\partial Q_{in}}{\partial m} = H_{LT} = H_{LM} + H_{Lm}$$

$$H_{LM} = f\frac{L\bar{V}^2}{2gD};\quad H_{Lm} = k\frac{\bar{V}^2}{2g}$$

$f_{\mathrm{LAM}} = 64/\mathrm{Re_D}$

$$\frac{1}{\sqrt{f}} \approx -1.8\log\left[\frac{6.9}{\mathrm{Re}_D} + \left(\frac{(\varepsilon/D)}{3.7}\right)^{1.11}\right]$$

$$u = u^*\left(B + \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right)\right)$$

where $u^* = \sqrt{\tau_{wall}/\rho}$,

$B = 5$, and $\kappa = 0.41$

$$D_H = \frac{4A}{Perimeter}$$

Dimensional Analysis

$\mathrm{Re} = \rho UL/\mu$

$\mathrm{Fr} = V^2/gh$

$C_D = F/(\tfrac{1}{2}\rho A V^2)$

Π theorem: $(n - m)$ Π groups.

Bernoulli

$p_0 = p_{\mathrm{dyn}} + p_{\mathrm{stat}}$

$p_{\mathrm{dyn}} = \tfrac{1}{2}\rho V^2$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Integral Conservation Laws

$$M = \int_V \rho\, dV$$

$$\int_{CM} \vec{F}_{\mathrm{external}}\, dt = \int \rho \vec{\mathbf{V}}\, dV$$

$$N = \int_{volume} \rho n\, dV$$

$$\dot{m} = -\int_A \rho \vec{V} \cdot d\vec{A}$$

$$Q = \int \vec{V} \cdot d\vec{A}$$

$$\dot{N} = -\int n\rho \vec{V} \cdot d\vec{A}$$

$$\left.\frac{dN}{dt}\right|_{CM} = \frac{d}{dt}\int_{CV} n\rho\, dV + \int n\rho \vec{V} \cdot d\vec{A}$$

$$0 = \frac{d}{dt}\int_{CV} \rho\, dV + \int \rho \vec{V} \cdot d\vec{A}$$

$$\Sigma F_{CV} = \frac{d}{dt}\int_{CV} \vec{V}_{xyz} \rho\, dV + \int \vec{V}_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A} + \int \vec{a}_{RF}\, \rho\, dV$$

$$\dot{Q}_{in} - \dot{W}_{\mathrm{net\ out}} = \frac{\partial}{\partial t}\int_{CV} e\rho\, dV + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\left(\vec{V} \cdot d\vec{A}\right)$$

Potential Flow

$\boldsymbol{\zeta} = \nabla \times \boldsymbol{V} = 2\boldsymbol{\omega}$

Streamlines: $dy/dx = v/u$

Irrotational, 2D, steady: $\nabla^2 \psi = 0$

$\psi = -\int v\, dx + f_1(y)$

$\psi = +\int u\, dy + f_2(x)$

$\Delta\psi = Q/b$

$\vec{V} = \vec{\nabla}\phi \rightarrow u = \partial\phi/\partial x,\ v = \partial\phi/\partial y$

Irrotational, steady: $\nabla^2 \phi = 0$

$\Gamma = \int \vec{\nabla} \times \vec{V}\, dA = \oint \vec{V} \cdot d\vec{s}$

$w(z) = \phi + i\psi$, where $z = x + iy$

$\dfrac{dw}{dz} = v_x - iv_y$

Lift: $L = w\rho U_\infty \Gamma_{\mathrm{TOTAL}}$

Differential Conservation Laws

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \vec{V}\right)$$

$$\vec{a}_{\mathrm{particle}} = \frac{\mathrm{D}V_{\mathrm{particle}}}{\mathrm{D}t} \equiv \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V} + \frac{\partial \vec{V}}{\partial t}$$

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \cdot \vec{\nabla}\right)\vec{V} = -\frac{1}{\rho}\vec{\nabla} p + \vec{g} + \frac{\mu}{\rho}\nabla^2 \vec{V}$$

Basic Frictionless Compressible Nozzle Information for Air

$c_p = 1005$ J/kgK $\quad$ $c_v = 718$ J/kgK $\quad$ $R = 287$ J/kgK $\quad$ $k = 1.40$

Functions valid everywhere on either side of a standing shock:

$$p = \rho RT \qquad a = \sqrt{kRT} \qquad v_{\lim} = \sqrt{2c_p T_0}$$

$$T^* = T_0\left(\frac{5}{6}\right) \qquad p^* = p_0\left(\frac{5}{6}\right)^{3.5} \qquad \rho^* = \rho_0\left(\frac{5}{6}\right)^{2.5} \qquad a^* = a_0\left(\frac{5}{6}\right)^{0.5}$$

$$T_0 = T\left(1+0.2Ma^2\right) \qquad p_0 = p\left(\frac{T_0}{T}\right)^{3.5} \qquad \rho_0 = \rho\left(\frac{T_0}{T}\right)^{2.5} \qquad a_0 = a\left(\frac{T_0}{T}\right)^{0.5}$$

Ma as a function of *A*: $\frac{1}{Ma}\left(\frac{\left(1+0.2Ma^2\right)^3}{1.728}\right) - \frac{A}{A^*} = 0$ $\qquad$ $A^*_{\max} = \frac{\dot{m}_{\max}\sqrt{RT_0}}{0.6847315\,p_0}$ $\qquad$ $\dot{m} = \rho A v$

Property changes immediately across the shock (1 → 2):

$$Ma_2^2 = \frac{2+0.4Ma_1^2}{2.8Ma_1^2 - 0.4} \qquad \frac{p_2}{p_1} = \frac{2.8Ma_1^2 - 0.4}{2.4} \qquad \frac{\rho_2}{\rho_1} = \frac{2.4Ma_1^2}{2+0.4Ma_1^2}$$

$$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \left(\frac{2.4Ma_1^2}{2+0.4Ma_1^2}\right)^{3.5} \cdot \left(\frac{2.4}{2.8Ma_1^2 - 0.4}\right)^{2.5} \qquad \frac{A_2^*}{A_1^*} = \frac{Ma_2}{Ma_1} \cdot \left(\frac{2+0.4Ma_1^2}{2+0.4Ma_2^2}\right)^3$$