

Examples of Flows and their Associated Complex Potentials, following White's *Fluid Mechanics* (5<sup>th</sup> edition)

$$z = x + iy$$

$$w = \phi + i\psi$$

$$dw/dz = V_x - iV_y$$

Description	$\bar{V}$	$w$	$\psi$	$\phi$
<b>Uniform flow</b> See page 265, 545.	$V_x = U, V_y = V$	$w = (U - iV)z$	$Uy - Vx$	$Ux + Vy$
<b>Simple source</b> of strength $m$ $m^2/s$ , located at $z_0$ . $m = Q/(2\pi b)$ if $m < 0$ , it is a "sink". See page 265, 545.	$V_r = m/r$ $V_\theta = 0$  origin is "singular"	$w = m \ln(z - z_0)$	$m\theta$	$m \ln(r)$
<b>"Irrotational" vortex (CCW)</b> , with center at $z_0$ . This vortex is fastest at its center. Vorticity at the origin $\rightarrow \infty$ , but is zero everywhere else. See page 265, 546.	$V_r = 0$ $V_\theta = k/r$	$w = -ik \ln(z - z_0)$  Use negative values of $k$ for CW rotation; $\Gamma = 2\pi k$	$-k \ln(r)$	$k\theta$
<b>Doublet</b> of strength $\lambda$ $m^3/s$ , located at $z_0$ . See page 543.	$u = -\lambda(x^2 - y^2)/(x^2 + y^2)^2$ $v = -2\lambda xy/(x^2 + y^2)^2$	$w = \lambda/(z - z_0)$	$-\lambda \sin(\theta)/r$	$\lambda \cos(\theta)/r$
<b>Stagnation Point</b> (corner flow), $u > 0$ along $y=0, x > 0$ . See page 546.	$n$ corresponds to the angle $\beta$ of the corner: $\beta = 180^\circ/n = \pi/n$	$w = Az^n$	$Ar^n \sin(n\theta)$	$Ar^n \cos(n\theta)$