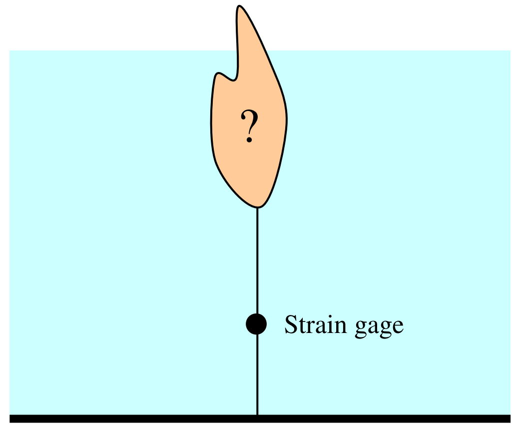
General requirements

1. Projects will be completed by groups of two or three students. Groups of one are not permitted. Each *group* will submit one copy of the project report. Although a list of potential projects is provided, you may create a project of your own instead. The projects below are mostly based on material from the book. Alternate projects may be computational, theoretical, or experimental, but are subject to approval by Dr. Pogo.
2. You may choose your own groups. You must notify Dr. Pogo of your group membership by Thursday, March 9, 2023. Any student not in a group by that date will be assigned to a group of two by Dr. Pogo, without your consent.
3. You must notify Dr. Pogo of your choice of project by Thursday, March 30, 2023. Your project choice, even if it is from the list below, requires approval from Dr. Pogo.
4. Projects are due before Thursday, May 18, 2021. Hardcopies are required. Accompanying electronic work must be submitted to my inbox.
5. Each project report should be professional and *self-contained*. That means that even though you are required to submit any Mathematica or Excel files that you created to solve your problem, your report should be written on the assumption that I will NOT look at those extra files. The following should be included in each report:
   a. A dated cover page, indicating all team members. This cover page should include a one paragraph abstract.
   b. An explanation of the problem that you are trying to solve. Remember that this report is to be self-contained. Do not refer to “the figure on the handout”, or “according to the book”, or other similar references.
   c. A *discussion* of the analysis *methods*. This should include a discussion of any assumptions that you make, including a justification for each assumption. That is, you may not merely state your assumptions, but you must discuss the extent to which each assumption is true, and the conditions required for each assumption to be considered valid. For example, the surface of a lake is NEVER flat, due to a combination of variable winds and surface tension, and the curvature and rotation of the earth. However, we commonly neglect all these effects when computing pressure at the lake bottom, because they do not significantly change the result. But, these assumptions might not be valid during a storm. To demonstrate whether something can be neglected, you **must** estimate *numeric values* for those things, and explain why these values are insignificant. Again: justifications must include both words and computations.
      As another example, we commonly neglect variations in water density when making computations. However, if the problem involves sound waves, then the small variations in density are the *most* important factor in computing the wave speed.
   d. Any pictures, plots, diagrams, tables, etc., that are necessary to communicate the problem, the solution, or your intermediate results. All figures should be textbook quality. Generally, the default plots made by both Mathematica and Excel are beyond inadequate with regard to labels, fonts and font sizes, colors, line thicknesses, aspect ratios, and a dozen other things. Mathematica printouts are also not of presentation quality if they include code (such as "Out[11] =" or "y[x_] := x;").
   e. A discussion of your *results*. This will differ for each project, and may include issues of cost, manufacturability, efficiency, ease of use, limitations on use, maintenance, environmental impact, and/or other items.

6. Text portions of the project must be typed, and equations must be professionally formatted. Symbolic calculations may not be written in either Mathematica notation or "Calculator notation" (3* x^2, or 3E-7). Don't even think about using copy-and-paste from Mathematica, because Mathematica is in Mathematica notation rather than presentation notation.
7. **Grading** will be a function of project difficulty and the number of students in the project. Easier projects will be more strictly graded, as will projects undertaken by groups of three students. For each suggested project, I have tried to include an estimate of the project difficulty, on a scale from 1 to 10.
8. You may not write in the imperative (like a lab manual). I suggest you look this word up if you don't know what it means. Also, your tense may be either past or present, but never future. If you have a figure, or table, etc., you *will* refer to it in your text somewhere. Also, the figure will have a name (e.g., "Figure 3"). See *any* textbook for examples of these rules.
9. Some of the projects will require some "library" work on your part. Every project has some ambiguities, and every project requires some knowledge of material that has not yet been, and may not ever be, covered in class.
10. You will not include any artwork that you didn't generate yourself. Images from the web are completely forbidden.
11. Comments on plots generated with Excel:
    a. If you have a *title*, it should be more than "y axis vs. x axis", or "V vs t", or "Speed vs. time". These are not very helpful. The title should indicate what is important about the plot. The title should use *words* to describe what these things are. See the caption of any plot in the text for an example (e.g., see figure 6.4, page 345 of the fluids textbook just to pick one at random).
    b. Don't use zillions of trailing zeroes in your axis values. For example, a time axis should not go from "0.0000" to "100.0000", etc. Use "100", or at most, "100.0", for the axis labels.
    c. Be careful with capitalization. As you know, most symbols change meaning when capitalized.
    d. Axis values should have a **range** that has significance to the problem. Both the minimum and maximum values plotted should be relevant (e.g., not $-\infty$ to ∞). Don't accept anything that Excel does for you without checking to see if it is what you really want. Trendlines, if used, will almost certainly require variables other than "y" and "x". Also, trendline slopes (etc.) have units, even though Excel doesn't tell you what they are.
    e. Along these lines, Excel adds a stupid little box ("series 1") onto the right hand side of all plots. Unless you are showing more than one set of data on a plot, this box should be deleted.
    f. Experimental data points are rarely connected by lines. Computed results often are. Make sure you know the difference between these two.
    g. Axis titles should indicate units.

Project #1: Reservoir level sensor

Difficulty factor: 5



You are required to measure the depth of a water reservoir, which typically varies between 5m and 8m. To do this, it is proposed that a cable be tied to the bottom of the reservoir, with the other end attached to a floating buoy. As the water level changes, a nonlinear force is created in the cable, which is converted into a voltage by a strain gage on the cable, which can be read on a monitor in a control room.

The *desired* relationship between depth and force in the mooring line is shown in the table.

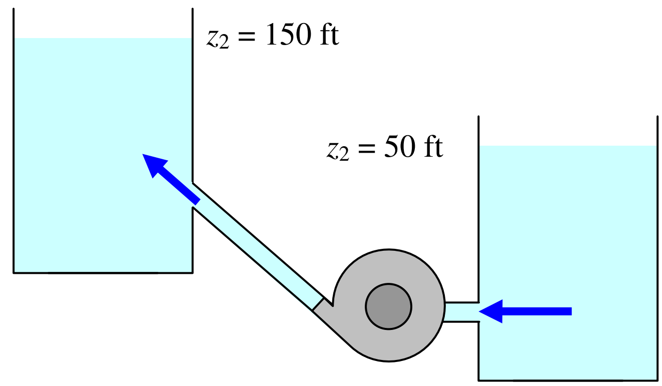
Depth (m)	Force (N)
5.00	210
5.50	312
6.00	400
6.50	472
7.00	530
7.50	573
8.00	600

Requirements:

1. Design the buoy. Specify materials, geometry, and tolerances. Be sure to include any necessary information about the mooring line (e.g., the length, the mass, etc.), including a way to connect it. The design should be of inexpensive materials and simple construction. The design should be sufficient that a third party could *completely* manufacture it without any further discussion with the designers. That is, you should include all mounting screws, etc. You may not neglect the weight of these screws, cables, etc. If you can't be bothered to do this kind of practical detail work, then pick another project.
2. Perform an analysis of the uncertainty in the reservoir depth reported by your system. This analysis should be extensive. For example, you should include a plot of the uncertainty vs. the reported depth.
3. In addition to the other elements in your discussion, you should evaluate the response of this system when the water depth is not in the five to eight meter range.
4. Because of the relative ease of this project, students choosing it do not usually earn the highest grades. To earn a high grade, you need to *completely* provide the information requested in (1), above. Specificity is needed… I should be able to take your report to any machine shop, hand it to them, and have them manufacture your device without them asking me a single question.

Project #2: Pump Selection

Difficulty factor: 4.5



We want to move water (at 20°C), from a gravitationally lower reservoir to a higher reservoir using a pump. In the pipe connecting the two reservoirs, the pipe friction losses are approximated by:

$$h_f \approx 27V_{\text{ave}}^2/(2g)$$

The connecting pipes all have a diameter of 6 inches. The pump is driven by an electrical motor, and the cost of the pump is proportional to the power input required.

The pump performance is described by $\phi = 6.04 - 161\zeta$, where

$$\phi = \frac{gh}{n^2 D_p^2}, \text{ and } \zeta = \frac{Q}{nD_p^3}.$$

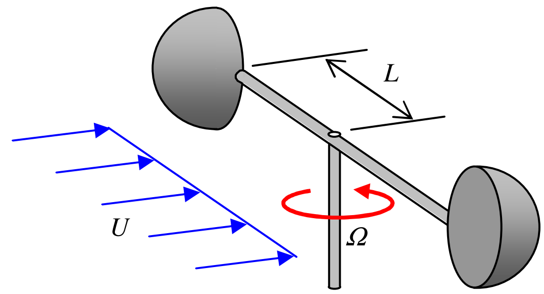
The efficiency of the pump is given by $\eta = 70\zeta - 91500\zeta^3$, and is defined as the power transmitted to the water divided by the power input to the pump form the motor.

> Q is the flow rate through the pump
> n is the rate of rotation of the shaft from the motor driving the pump.
> H is the pump head, in feet.
> D_p is the diameter of the impeller (the pump "fan" blades)
> $\rho g Q h$ is the power transmitted to the water.

The parameter ζ is valid between $0.000 < \zeta < 0.027$ only.

The pump must rotate no slower than 600 rpm, and deliver no less than 1 ft^3/s of water. Select a *low-cost* pump and specify its operating conditions. That is, choose values of D_p, Q, and n for a "low cost" pump. In addition, determine the efficiency of your chosen pump. Show that the cost per flow rate of your pump is low compared to other choices.

Project #3: Simple Anemometer



Difficulty Factor: 3

Our textbook provides the following data for a two-cup anemometer. The anemometer is made from a split ping-pong ball (d = 1.5 in) glued to a thin ($^1/_4$ inch) rod pegged to a center axle. Experiments were done in a wind tunnel, using four different lengths of the rods.

The experimental results for average rate of rotation are as follows (data was taken at 1 atmosphere and 20 °C):

L = 0.212 ft		L = 0.322 ft		L = 0.458 ft		L = 0.574 ft	
U (ft/s)	Ω (rpm)	U (ft/s)	Ω (rpm)	U (ft/s)	Ω (rpm)	U (ft/s)	Ω (rpm)
18.95	435	18.95	225	20.10	140	23.21	115
22.20	545	23.19	290	26.77	215	27.60	145
25.90	650	29.15	370	31.37	260	32.07	175
29.94	760	32.79	425	36.05	295	36.05	195
38.45	970	38.45	495	39.03	327	39.60	215

The angular velocity Ω might be a function of wind speed, air density, air viscosity, rod length, and cup diameter.

Define all the appropriate dimensionless Π groups for this problem, and plot the given data in the appropriate dimensionless manner. Analyze the *uncertainty* of the results.

Use this geometry to design a large scale anemometer for an airport, with $d \approx$ 30cm. If wind speeds vary (the max is 25 m/s), and we desire an average rotation rate of Ω = 120 rpm (why?), what is the proper length of the rods, L? Plot the resulting expected value of Ω as a function wind speed. What are the limitations of this design?

Again, note the low difficulty factor for this project, suggested that not only perfection of computation but also perfection in presentation are required for a merely reasonable good grade.

Project #4: Motion of a golf ball

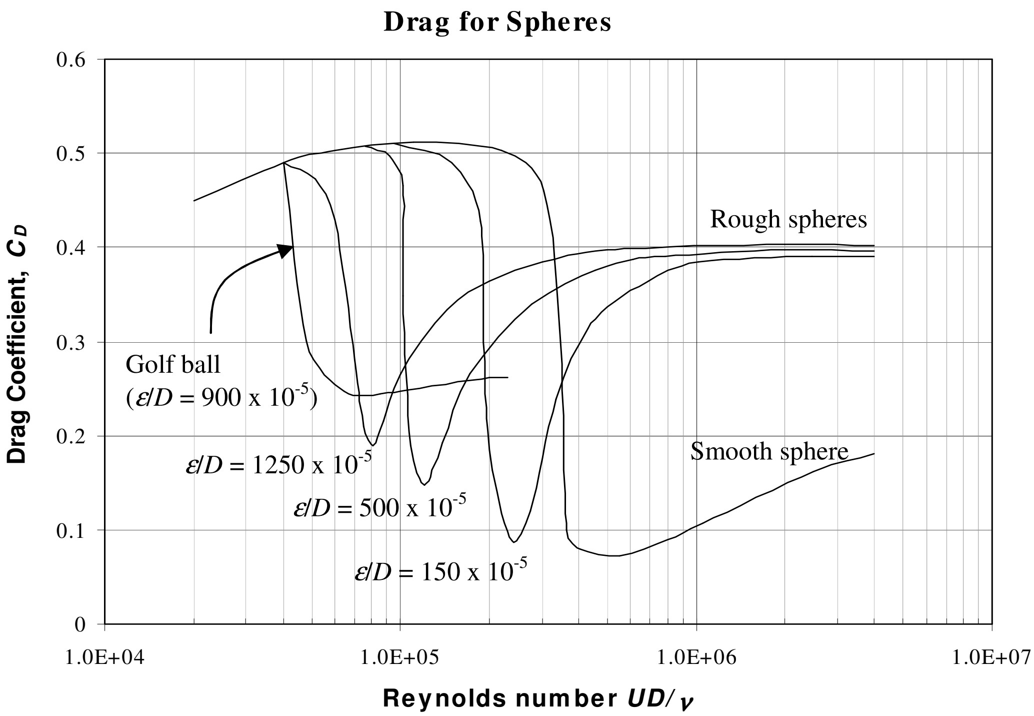
Difficulty Factor: 8

For a certain range of speeds, the degree of roughness of an object can have a strong impact on the drag of the object in flight. Ultimately, this is why golf balls have dimples (roughness). Some experimental data for "drag" as a function of Reynolds Number are given below. It is easy to see that at certain speeds, a rough (dimpled) ball has less drag than a smooth ball, and should go further when given an initial velocity. The drag coefficient and Reynolds Number are defined on page 311 of our text. Drag is seen to be a function of the relative roughness of a sphere (ε/d), where ε is the average depth of a "valley" on the surface of the sphere, and d is the diameter of the ball. The figure is based on figure D5.2 in the book (page 338).

Also, topspin and underspin affect the driving distance. Bernoulli's equation can be used to help you calculate the additional force on a spinning object.

The mass of a golf ball is about 46g, and the diameter is about 4.3 cm. Assuming standard pressure and temperature, compare the trajectory of golf balls, roughened spheres, and smooth spheres for a reasonable range of initial conditions (initial velocity, initial angle, absolute height above or below the green, moderate wind, degree of topspin or underspin). You should be able to answer such questions as: under what conditions can a rough sphere outfly a real golf ball? How does roughness affect an average golfer, compared to a pro? Under what conditions does a tailwind hurt your distance?

You might also check out the paper "Data Correlation for Drag Coefficient for Sphere" by Faith A. Morrison.



Project #5: Municipal Water Tower

Difficulty Factor: 6

We want to design a system to keep a one million gallon capacity water tank filled. The plan is to use a modified version of the model 1206 centrifugal pump (manufactured by Taco Inc, Cranston, RI). Test data for a smaller *scale model* of this pump (having D = 5.45 in; Ω = 1760 rpm; water at 20°C) are provided.

Q (gal/min)	0	5	10	15	20	25	30	35	40	45	50	55	60
H (ft)	28	28	29	29	28	28	27	26	25	23	21	18	15
η (%)	0	13	25	35	44	48	51	53	54	55	53	50	45

The tank is to be filled with cool groundwater (10°C) from an aquifer, which is 0.8 miles from the tank, and 150 ft lower than the tank. The estimated daily water use is 1.5 million gallons per day; filling time should not exceed 8 hours per day.

Because of other needs, the piping system will also have four valves and 10 elbows between the aquifer and the tank. During operation of the system, you may consider that the valves are completely open. The piping will be galvanized iron.

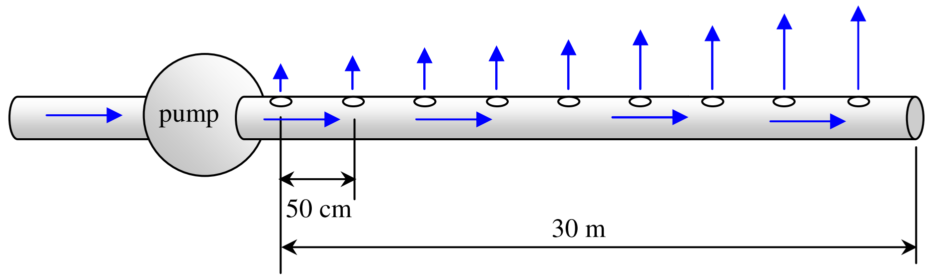
Your objective is to select an *economical* pipe size, pump impeller size (D), and pump speed for this task. To do this, examine the pump-test data in *non-dimensional form*. The final system should be economical both in terms of capital costs, and operating expense. Although one system could be cheaper to buy, it could be more expensive to operate.

The following cost estimates are available:

Pump and motor:	\$3500 plus \$1500 per inch of impeller
Pump speed:	between 900 and 1800 rpm
Valves:	\$300 + \$200 per inch of pipe size
Elbows:	\$50 + \$50 per inch of pipe size
Pipes:	\$1 per inch of diameter per foot of length
Electricity:	\$0.17 per kilowatt-hour

Project #6: Irrigation System

Difficulty Factor: 7



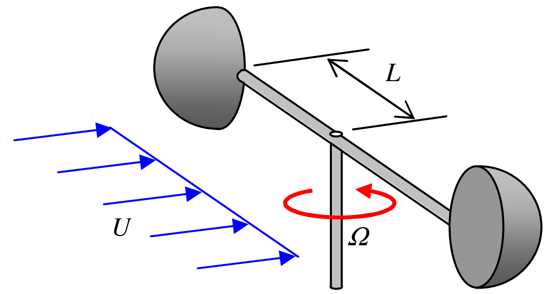
A commercial hydroponic garden is irrigated with water at 20°C by a 30 m long perforated pipe (the figure is not to scale). The pipe is 10 cm in diameter, and there is a circular hole every 50 cm. A pump delivers water at 75 kPa (gage) to the entrance, and the other end of the pipe is sealed. Experience and theory tell us that near the closed end of the pipe, the pressure is much higher than in the middle of the pipe, so that there will be a relatively large amount of flow through the last holes compared to the early holes. Friction **is significant** in this problem. Assume that the holes are on the side, not the top, of the pipe (so, the picture shown is a top view).

We prefer that the volumetric flow through each hole be identical. One method to do this is to vary the size of the holes along the pipe axis. Determine a distribution of hole diameters that will make the flow distribution as uniform as possible. Of course, the holes may not be any arbitrary size; they should correspond to commonly available (metric) drill bit sizes.

You should show why your design is optimum, and you should predict the flow rate through each hole.

Project #7: Advanced Anemometer

Difficulty Factor: 9



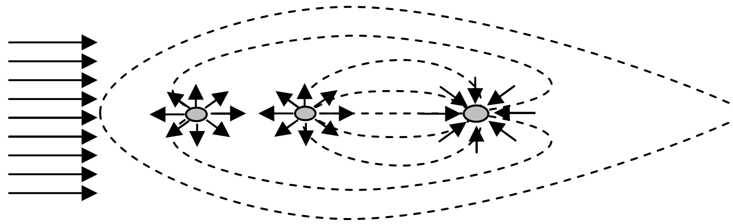
We want to build/design an anemometer that is a little more sophisticated than the one described in project #3 (which utilized an "average torque" method). In reality, the angular speed Ω is a *function* of how far the cups are turned into the wind, and varies at every instant, even for a constant wind speed.

You should modify the design so that there is an approximately linear relationship between wind speed and rotation rate, at least when wind speeds U are between 20 and 40 mph. The anemometer should rotate at about 360 rpm when U = 30 mph. You must select values for cup size and rod length. You must select materials for all components. You should select a commercially available bearing, and include the appropriate bearing friction in your calculations.

You can start by estimating the *instantaneous* drag of the cups as a function of the rotation angle $\theta(t)$ of the system.

Project #8: Symmetric Wing Modeling using Potential Flow

Difficulty Factor: 7.5



In 1927, Theodore von Kármán developed a superposition method to mathematically model physical objects, such as wings, with a series of fluid sources and sinks in a uniform flow. In such a model, the mathematically derived streamlines can, by appropriately choosing the amount of flow through each source or sink, be made to mimic the streamlines of a real flow. As seen in the accompanying sketch, if the net amount of inflow and outflow through the sinks is the same, then there will be some closed streamline surrounding all of the sources and sinks. The flow *outside* of this closed streamline represents the frictionless flow that would occur around a wing having the same shape as the closed streamline. The accompanying sketch shows a 2D shape that might occur if you had two sources and one sink, where the sink "drains' fluid out at the same rate that the two sources (combined) bring fluid in (see sections 4.10 and 8.3 of our text for more information).

You must write a computer model (in C, BASIC, Excel, Mathematica, whatever) to determine the 2D locations and strengths (amount of flow) for the various sources and sinks necessary (it will certainly require more than 20) to model the specific airfoil below. Compare your results (*velocities* and *pressures*) to the theoretical results presented in "Theory of Wing Sections", I.H. Abbott and A.E. von Doenhoff, Dover, New York, 1959 (I have a copy). You should aim for your results to agree within 1%. Since this wing is symmetric, all the sources and sinks should be along the axis of the body. You probably need more sources/sinks near the nose and tail.

The shape of the NACA 0018 wing (y as a function of x) is given by:

$$\frac{y}{t_{\max}} = 1.4845\zeta^{1/2} - 0.63\zeta - 1.758\zeta^2 + 1.4215\zeta^3 - 0.5075\zeta^4$$

where $\zeta = x/C$, and C is the overall length of the foil along the x axis (the wing goes from ζ=0 to ζ=1). For this shape, use $t_{\max} = 0.18$. This maximum thickness occurs at $\zeta = 0.3$.

Some of the required plots include:

a. A streamlines plot of the flow field near the wing (before, above, behind).
b. A plot of source strength vs. ζ.
c. A plot of source position vs. ζ.
d. A plot of the pressure field near the wing.
e. Plots of v_{computed} and p_{computed} on the surface vs. the experimental results in *Abbott*.

Project #9: Wing Modeling using Potential Flow with Lift

Difficulty Factor: 9

First, see the description for project #8. A perfectly symmetric wing has no lift, but real wings are typically not vertically symmetric. They have both an angle of attack, and a non-symmetric shape. To account for this, vortices must be added to the model, as well as the sources and sinks described above. The total lifting force of the wing can then be computed from the total strength of the vortices you include.

The streamlines in the solution should be parallel to the wing at all points. Additionally, the "Kutta" condition must be satisfied, which means that the rear stagnation point must be exactly at the tail of the wing.

This wing is defined by:

$$y^*_{\text{top}} = 2.6x^* - 6.5x^{*2} + 6.3x^{*3} - 2.7x^{*4}$$
$$y^*_{\text{bottom}} = -1.18\sqrt{x^*} + 1.5x^* - 0.62x^{*2}$$

Use $x^* = x/L$, and $y^* = y/w$, where $w = 0.15L$. The wing defined by these equations has an angle of attack of about 2.58°.

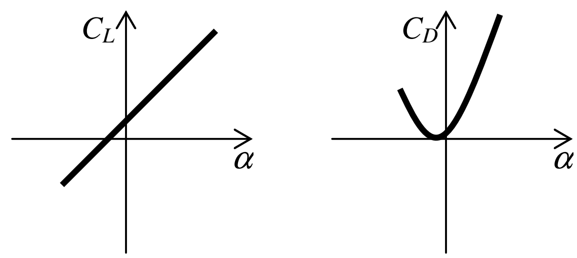
Project #10: Wing Modeling using Finite Elements

Difficulty Factor: 9

The behavior of a wing can also be modeled using Finite Elements in Excel, as we did in class. Every grid element that represents a part of the cross section of the wing has the same value of ψ, although you can't know ψ *a priori*. Grid values are iterated until the Kutta condition is satisfied (the rear stagnation point must be exactly at the tail of the wing, and the streamline there must aim away from the wing at the same angle as the wing).

Once ψ is known for each cell, use Bernoulli's equation to compute the pressure in each cell, and integrate these over the contour of the wing to find the net upwards force (Lift), and the net horizontal force (Drag).

Determine the lift and drag as functions of the angle of attack α of the wing, for $-10° \le \alpha \le +20°$. What are the lift and drag forces at zero angle of attack? The coefficients of lift and drag should appear somewhat as follows:



As with project 9, use a wing that is defined by:

$$y^*_{\text{top}} = 2.6x^* - 6.5x^{*2} + 6.3x^{*3} - 2.7x^{*4}$$

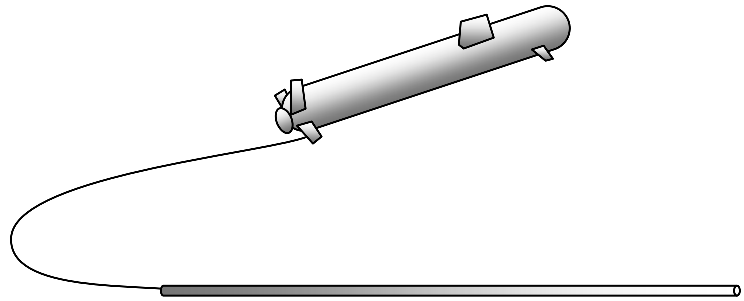
$$y^*_{\text{bottom}} = -1.18\sqrt{x^*} + 1.5x^* - 0.62x^{*2}$$

Use $x^* = x/L$, and $y^* = y/w$, where $w = 0.15L$. The wing defined by these equations has an angle of attack of about 2.58°.

Compare you results to those found for any similar wing in Abbott (see reference in Project #8).

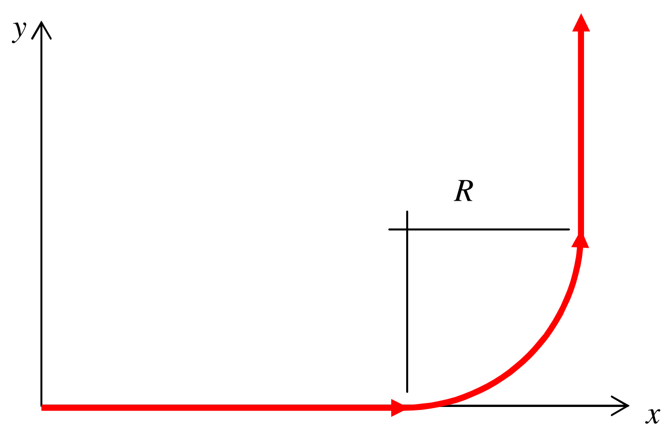
Project #11: Horizontal Single-Element Towed Array Sonar

<u>Difficulty Factor</u>: 11



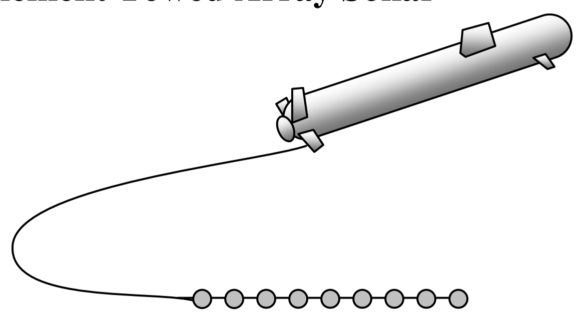
Submarines often drag behind them a horizontal “towed array” of sonar detectors to detect other vessels. When deployed, the array can be modeled as a long rigid cylinder, perhaps of length L = 800m and diameter D = 9 cm. The front of the array is connected to a flexible cable (with a length of perhaps a = 400m), connected to the tail of the sub. The sub might be both moving and maneuvering, and it is desired to know the orientation and position of the array as a function of time. Consider a sub traveling with constant speed V that eventually turns through a radius R, as sketched below. You’ll have to estimate reasonable values of V, R, and the mass of the array. Be very cautious: there is only one possible correct estimate for mass!

As both the sub and array accelerate, determine the position and orientation of the array. What is the maximum tension in the cable? How much time is required before the array is once again parallel to the motion of the sub? In this case, what criteria is used to define “parallel”? What if the sub makes a turn of less than 90°? How much time is required for the array to stop after the sub comes to a complete stop? Neglect viscous forces on the cable itself, and focus on the forces on the array.



Position of the tail of the sub

Project #12: Horizontal Multiple-Element Towed Array Sonar



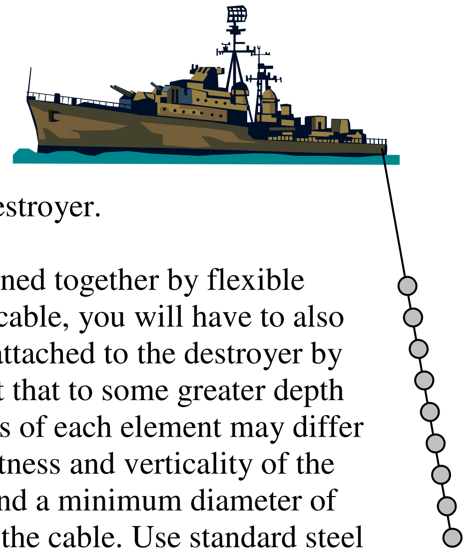
Difficulty Factor: 10

First, see the description for project 11. Then, investigate this problem:

Instead of a single, large cylinder, the array could more accurately be modeled as a series of identical spherical elements joined together by flexible cables. In addition to considering the tension in each part of the cable, you will have to also consider the “straightness” of the array as a group. How will you measure straightness?

Project #13: Vertical Towed Array Sonar

<u>Difficulty Factor</u>: 8



First, see the description for projects 11 and 12. Whale researchers use *vertical* acoustic arrays, and surface vessels searching for submarines might also use a vertical towed sonar array. Consider an array of spherical elements towed behind a destroyer.

This array could be modeled as a series of spherical elements joined together by flexible cables. In addition to considering the tension in each part of the cable, you will have to also consider the “straightness” of the array as a group. The array is attached to the destroyer by some length of flexible cable (suppose 100m), and descends past that to some greater depth (suppose 300m). There are to be at least 20 spheres, and the mass of each element may differ from the others, in order to attempt to maximize both the straightness and verticality of the array. However, each element has a minimum mass of 200 kg, and a minimum diameter of 1.0m. Also, increasing the mass will also increase the tension in the cable. Use standard steel cable with a diameter of 1 inch. Specify the mass and diameter of each spherical element if the destroyer travels at 10 knots. How will you define or measure straightness and verticality? What is the safety factor on the support cable?

Project #14: Channel Flow Velocity Profile

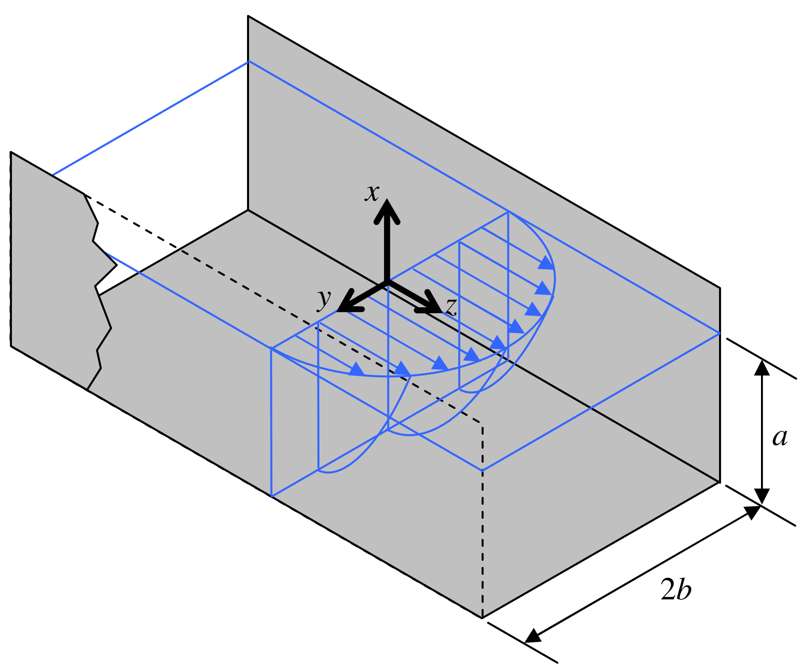
Difficulty Factor: 8

Flow in rivers and flumes are similar to each other, although one is gravity driven and the other is pressure driven. Assuming that the flow is one dimensional, determine the velocity profile $w(x, y)$ for several different flows having known width $2b$, depth a, and flow rate Q. For this problem, the Navier-Stokes equations reduce to a form known as the "Poisson Equation", which is just a little harder to solve than Laplace's Equation. The coordinate origin shown is at the point of highest speed (centered in y, and at the top of the flow). The flow is caused by a slight downward incline θ.

The following "solution" satisfies all the necessary boundary conditions, but does *not quite* satisfy the Navier-Stokes solution.

$$w(x, y)_{\text{parabolic approximation}} = \frac{9Q}{8a^3b^3}\left(x^2 - a^2\right)\left(y^2 - b^2\right)$$

1. Solve the problem symbolically. Your answer will be a series solution in terms of pressure gradients or gravity components rather than Q. Use the solution to determine the value of Q that corresponds to your chosen geometry and pressure gradient (or gravity component).
2. Solve the problem numerically. There exists no universe in which the preceding sentence could reasonably mean "let Mathematica do this problem for you". YOU must solve it. That means that YOU must make decisions about numerical modeling (including, among others, the technique you use to approximate derivatives). You must explain these decisions. I recommend Excel rather than Mathematica.
3. Find sensible ways to compare the three solutions.



Project #15: 2D Flow over a step with viscosity and separation.

Difficulty Factor: 10

We created a finite-element model in Excel this semester to solve a potential flow version of the 2D flow over a backwards-facing step in class this semester. However, that model did not include viscosity. When viscosity is included, it is *much* more difficult to solve this problem. When viscosity is included, then the results depend strongly on the Reynold's Number. If Re is very low, then the solution looks very much like the potential flow solution. However, as Re increases, a region of flow separates from the main flow, as shown in this sketch. The incoming flow on the left is parabolic.

Use Mathematica or Excel to solve the (non-dimensionalized) Navier Stokes equations (mass, x momentum, and y momentum) for the three fields u, v, and p. Determine the size of the separated zone as a function of Re. Compare the results to published values. Include plots of p and vorticity. You should also include some Mathematica "streamplots", too.

You might see the paper "Direct numerical simulation of turbulent flow over a backward-facing step", by Michal A. Kopera, Robert M. Kerr, Hugh M. Blackburn, and Dwight Barkley for a discussion of how a 3D solution to this problem would be created. Your solution would be much simpler, having only two dimensions and no turbulence.