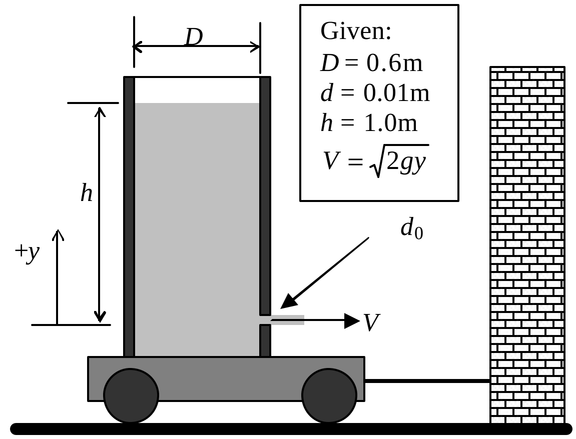

Recall this problem from class:

Using the integral form of momentum conservation, we discovered that the tension in the rope was:

$$\boxed{T = \frac{1}{2} rgpd^2 y}.$$

We could also apply the mass conservation equation to this problem:

$$0 = \frac{d}{dt}\int_{CV} r dV + \int r\vec{V} \cdot d\vec{A}\,.$$

Factoring out r, and substituting for the known $\vec{V}$:

$$0 = r\left(\frac{d}{dt}\int_{CV} dV + \sqrt{2gy}\,\frac{pd^2}{4}\right)$$

$$\frac{d}{dt}\int_{CV}\frac{pD^2}{4}dy = \frac{pD^2}{4}\frac{d}{dt}\int_{CV}dy = \frac{pD^2}{4}\frac{dy}{dt} = -\sqrt{2gy}\,\frac{pd^2}{4} \rightarrow \frac{dy}{dt} = -\sqrt{2gy}\,\frac{d^2}{D^2}\,.$$

This is a separable first order differential equation: $\boxed{y^{-\frac{1}{2}}dy = -\sqrt{2g}\,\frac{d^2}{D^2}dt}$.

Integrating both sides: $2y^{\frac{1}{2}}\Big|_{y=h}^{y} = -\sqrt{2g}\,\frac{d^2}{D^2}t\Big|_0^t \rightarrow 2y^{\frac{1}{2}} - 2h^{\frac{1}{2}} = -\sqrt{2g}\,\frac{d^2}{D^2}t$

Solving for y: $\boxed{y(t) = \left(h^{\frac{1}{2}} - \sqrt{\frac{g}{2}}\,\frac{d^2}{D^2}t\right)^2}$.

This could be combined with the above expression for tension: $\boxed{T(t) = \frac{1}{2} rgpd^2\left(h^{\frac{1}{2}} - \sqrt{\frac{g}{2}}\,\frac{d^2}{D^2}t\right)^2}$.

The depth as a function of time is plotted here: