

1. [2pt] A cue stick hits a cue ball with an average force of 26 N for a duration of 0.027 s. If the mass of the ball is 0.16 kg, how fast is it moving after being struck?

2. [2pt] A rookie baseball catcher requires only 1.25 ms to catch a fastball of mass 150 g that is moving at a speed of 130 km/h. On the other hand, a veteran catcher moves his hand backwards during the catch, increasing the stopping time to 10.50 ms. How much more force does the rookie catcher experience than the veteran?

3. [2pt] An object of mass 3.9 kg is projected into the air at a 45° angle. It hits the ground 3.8 s later. What is the magnitude of its change in momentum while it is in the air? Ignore air resistance.

4. [2pt] Within cells, small organelles containing newly synthesized proteins are transported along microtubules by tiny molecular motors called kinesins. What force does a kinesin molecule need to deliver in order to accelerate an organelle having mass 0.026 pg from rest to 2.4 $\mu\text{m/s}$ over a time of 30.0 μs ?

5. [3pt] A tennis ball of mass 0.065 kg is served. It strikes the ground with a velocity of 54 m/s at an angle of 24° below the horizontal. Just after the bounce it is moving at 51 m/s at an angle of 17° above the horizontal. If the interaction with the ground lasts 0.058 s, what average force did the ground exert on the ball?

6. [2pt] A small asteroid ($m_{tot} = 6750$ kg), flying through space with a speed of 300 m/s, cracks into two pieces as a result of thermal stresses. The first piece has a mass of 3800 kg and continues in the same direction as the original asteroid, but with a speed of 485 m/s. What is the speed of the second piece?

7. [2pt] A firecracker ($m_{tot} = 0.65$ kg), sitting on a frozen pond, explodes into three pieces, each of which moves horizontally. Piece 1 ($m_1 = 0.20$ kg; $v_1 = 90$ m/s) moves at a right angle to piece 2 ($m_2 = 0.25$ kg; $v_2 = 115$ m/s). What is the speed of the third piece?

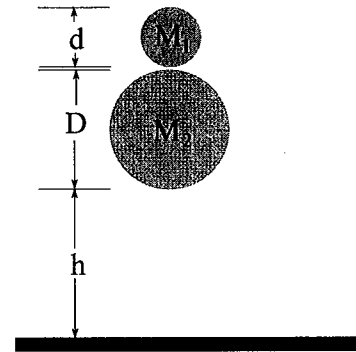
8. [2pt] A stationary 0.190 gram fly encounters the windshield of a 1175 kg automobile traveling at 106 km/h, and sticks to it. What is the change in speed of the car due to the fly?

9. [2pt] Kevin has a mass of 86.6 kg and is skating with in-line skates. He sees his 21.4 kg younger brother up ahead standing on the sidewalk, with his back turned. Coming up from behind, he grabs his brother and rolls off at a speed of 2.42 m/s. Ignoring friction, find Kevin's speed just before he grabbed his brother.

10. [2pt] A baseball, moving at a speed of 35 m/s, is struck by a bat moving towards it with an average speed of 23 m/s. The collision is elastic. Assuming that the mass of the bat is *much* larger than the mass of the ball, what is the final speed of the ball?

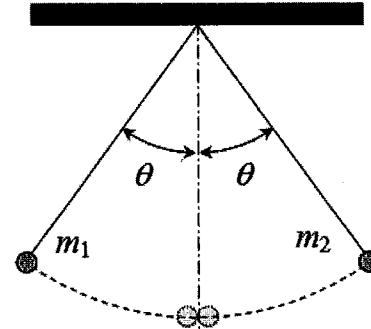
11. [2pt] By what factor did the kinetic energy of the ball increase?

12. [3pt] Two spheres of mass M_1 and M_2 are arranged one above the other as shown. They are separated by a fraction of a mm. They are released from rest and allowed to fall to the ground, a distance $h = 7.0$ m below. Mass M_2 collides elastically with the ground and then elastically with mass M_1 . Calculate the maximum height the center of M_1 rises above the ground after the collision. $D = 0$ cm, $d = 0$ cm, $M_1 = 0.12$ kg, $M_2 = 1.80$ kg.



13. [2pt] Small masses m_1 ($m_1 = 40$ kg) and m_2 ($m_{2A} = 50$ kg; $m_{2B} = 25$ kg; $m_{2C} = 40$ kg; $m_{2D} = 70$ kg; $m_{2E} = 15$ kg) are each attached to a string of length 1.2 m. The other end of each string is attached to a common point on the ceiling. The masses are raised until each string is at an angle of 33° with respect to vertical, and then simultaneously released. They collide elastically when the strings are vertical. For each case, rank the final velocity of m_2 , from smallest to largest. For example:

$D < A < C < B < E$

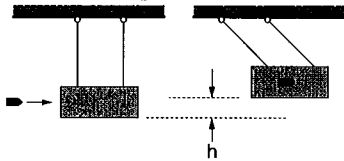


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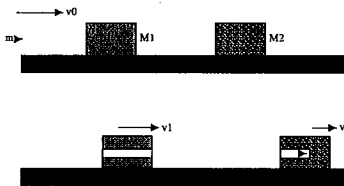
14. [3pt] Two 19.0 kg ice sleds are placed a short distance apart, one directly behind the other, as shown in the figure. A 4.44 kg cat, initially standing on sled 1, jumps across to sled 2 and then jumps back to sled 1. Both jumps are made at a horizontal speed of 2.22 m/s relative to the ice. What is the final speed of sled 2? (Assume the ice is frictionless.)



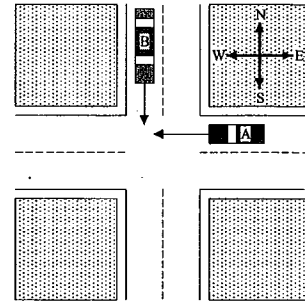
15. [2pt] A ballistic pendulum, as shown in the figure, was a device used in the past century to measure the speed of bullets. The pendulum consists of a large block of wood suspended from long wires. Initially, the pendulum is at rest. The bullet strikes the block horizontally and remains stuck in it. The impact of the bullet puts the block in motion, causing it to swing upward to a height h . If the bullet has a mass of 8.1 g, and the block of mass 3.2 kg swings up to a height of $h = 21.000\text{ cm}$, what was the speed of the bullet before impact?



16. [3pt] A bullet with a mass of 6.60 g is fired horizontally at two blocks resting on a smooth and frictionless table top as shown in the Figure. The bullet passes through the first 1.70 kg block, and embeds itself in a second 1.65 kg block. Speeds $v_1 = 3.40\text{ m/s}$ and $v_2 = 1.80\text{ m/s}$, are thereby imparted on the blocks. The mass removed from the first block by the bullet can be neglected. Find the initial speed of the bullet.



17. [2pt] Two vehicles A and B are traveling west and south, respectively, toward the same intersection where they collide and lock together. Before the collision A (total weight 1020 N) is moving with a speed of 46 m/s and B (total weight 1750 N) has a speed of 25 m/s. Find the magnitude of the velocity of the interlocked vehicles after the collision.



18. [2pt] What is the direction of motion of the interlocked vehicles after the collision? Give your answer in terms of the angle that the velocity vector makes with respect to East. (East = 0°, North = 90°, West = 180°, and South = 270°)

① Given:

$$F_{ave} = 26 \text{ N}$$

$$\Delta t = 0.027 \text{ s}$$

$$m_{ball} = 0.16 \text{ kg}$$

$$v_i = 0$$

$$\Sigma F_x \Delta t = \Delta p_x$$

$$F_{ave} \Delta t = m v_f - m v_i$$

$$\boxed{v_f = \frac{F_{ave} \Delta t}{m}} = 4.39 \text{ N}$$

② $\Delta t_r = 1.25 \text{ ms} = 1.25 \times 10^{-3} \text{ s}$

$$m = 150 \text{ g} = 0.15 \text{ kg}$$

$$v_{ball} = 130 \text{ km/hr} = 36.1 \text{ m/s}$$

$$\Delta t_v = 10.5 \text{ ms} = 1.05 \times 10^{-2} \text{ s}$$

$$\Sigma F_r \Delta t_r = \Delta p = m v_f - m v_i$$

$$F_r \Delta t_r = m(0 - v_{ball})$$

$$F_r = \frac{-m v_{ball}}{\Delta t_r}$$

similarly

$$F_v = \frac{-m v_{ball}}{\Delta t_v}$$

How much more force? $|F_r| - |F_v|$

$$|\Delta F| = \frac{m v_{ball}}{\Delta t_r} - \frac{m v_{ball}}{\Delta t_v} = m v_{ball} \left(\frac{1}{\Delta t_r} - \frac{1}{\Delta t_v} \right)$$

$$\boxed{|\Delta F| = m v_{ball} \left(\frac{\Delta t_v - \Delta t_r}{\Delta t_r \Delta t_v} \right)} = 3.82 \text{ N}$$

(3) Given:

$$m = 3.9 \text{ kg}$$

$$\theta = 45^\circ$$

$$\Delta t = 3.8 \text{ s}$$

$$\sum F_x \Delta t = \Delta P_x$$

$$\sum F_y \Delta t = \Delta P_y$$

$$0 = \Delta P_x$$

$$-mg \Delta t = \Delta P_y$$

$$|\Delta P| = \sqrt{(\Delta P_x)^2 + (\Delta P_y)^2}$$

$$|\Delta P| = \sqrt{(-mg \Delta t)^2}$$

$$\boxed{|\Delta P| = mg \Delta t} = (145 \text{ kgm/s})$$

(4) Given:

$$m = 0.026 \text{ pg} = 0.026 \times 10^{-12} \text{ g} = 2.6 \times 10^{-17} \text{ kg}$$

$$v_i = 0$$

$$v_f = 2.4 \text{ } \mu\text{m/s} = 2.4 \times 10^{-6} \text{ m/s}$$

$$\Delta t = 30 \text{ } \mu\text{s} = 3 \times 10^{-5} \text{ s}$$

$$\sum F \Delta t = \Delta p = m(v_f - v_i)$$

$$\sum F \Delta t = m v_f - 0$$

$$\boxed{F = \frac{m v_f}{\Delta t}} = (2.08 \times 10^{-18} \text{ N})$$

5) $m = 0.065 \text{ kg}$

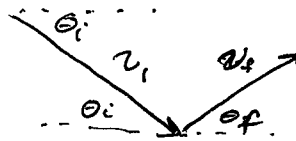
$v_i = 54 \text{ m/s}$

$\theta_i = 24^\circ$

$v_f = 51 \text{ m/s}$

$\theta_f = 17^\circ$

$\Delta t = 0.058 \text{ s}$



$v_{ix} = v_i \cos \theta_i$

$v_{fx} = v_f \cos \theta_f$

$v_{iy} = -v_i \sin \theta_i$

$v_{fy} = v_f \sin \theta_f$

$\Sigma F_x \Delta t = \Delta p_x = m v_{fx} - m v_{ix}$

$\Sigma F_y \Delta t = \Delta p_y = m v_{fy} - m v_{iy}$

$F_x = \frac{m(v_f \cos \theta_f - v_i \cos \theta_i)}{\Delta t}$

$F_y = \frac{m(v_f \sin \theta_f - (-v_i \sin \theta_i))}{\Delta t}$

$F_y = \frac{m(v_f \sin \theta_f + v_i \sin \theta_i)}{\Delta t}$

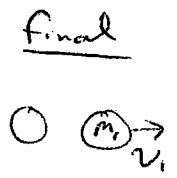
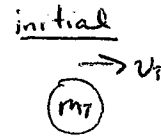
$|F| = \sqrt{F_x^2 + F_y^2} =$

$\frac{m}{\Delta t} \sqrt{(v_f \cos \theta_f - v_i \cos \theta_i)^2 + (v_f \sin \theta_f + v_i \sin \theta_i)^2}$

Given: $M_{\text{TOT}} = 6750 \text{ kg}$ $v_f = 300 \text{ m/s}$

$m_1 = 3800 \text{ kg}$ $v_1 = 485 \text{ m/s}$

$\vec{P}_i + \Sigma \vec{F} \Delta t = \vec{P}_f$



There are no external forces so $\Sigma \vec{F} \Delta t = 0$

$P_{ix} = M_T v_T$ $P_{fx} = m_1 v_1 + m_2 v_2$

$M_T v_T = m_1 v_1 + m_2 v_2$

but $m_2 = M_T - m_1$

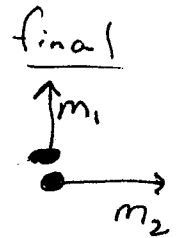
$m_2 v_2 = M_T v_T - m_1 v_1$

$v_2 = \frac{M_T v_T - m_1 v_1}{M_T - m_1}$

$= (6.17 \text{ m/s})$

⑦ Given: $M_{TOT} = 0.65 \text{ kg}$
 $m_1 = 0.20 \text{ kg}$ $v_1 = 90 \text{ m/s}$
 $m_2 = 0.25 \text{ kg}$ $v_2 = 115 \text{ m/s}$

initial



$$\vec{P}_i + \cancel{\Sigma \vec{F} \Delta t} = \vec{P}_f$$

$$\vec{P}_i = 0$$

x-components

y-components

$$0 = m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x}$$

$$0 = m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y}$$

$$v_{3x} = - \left(\frac{0 + m_2 v_2}{m_3} \right)$$

$$v_{3y} = - \left(\frac{m_1 v_{1y} + 0}{m_3} \right)$$

$$v_{3x} = \frac{-m_2 v_2}{m_3}$$

$$v_{3y} = \frac{-m_1 v_1}{m_3}$$

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \boxed{\frac{1}{m_3} \sqrt{(m_2 v_2)^2 + (m_1 v_1)^2}}$$

$$m_3 = M_{TOT} - m_1 - m_2$$

$$v_3 = \frac{1}{M_{TOT} - m_1 - m_2} \sqrt{(m_2 v_2)^2 + (m_1 v_1)^2} = \boxed{169.6 \text{ m/s}}$$

⑧ Given

$$m_f = 0.19 \text{ g} \quad v_f = 0$$

$$m_c = 1175 \text{ kg} \quad v_c = 106 \text{ km/h}$$

bewary of units!

There is an inelastic collision...

$$m_f v_f + m_c v_c = (m_f + m_c) v$$

$$v = \frac{m_f v_f + m_c v_c}{m_f + m_c} = \frac{m_c v_c}{m_f + m_c}$$

$$\Delta v = v - v_c$$

final velocity of car

initial velocity of car

$$\Delta v = v - v_c = \frac{m_c}{m_f + m_c} v_c - v_c = v_c \left(\frac{m_c}{m_f + m_c} - 1 \right)$$

$$= v_c \left(\frac{m_c}{m_f + m_c} - \frac{m_f + m_c}{m_f + m_c} \right) = -v_c \left(\frac{m_f}{m_f + m_c} \right)$$

$$= -1.71 \times 10^{-5} \text{ km/hr}$$

⑨ Given: $m_k = 86.6 \text{ kg}$ $v_{ki} = ?$

$m_b = 21.4 \text{ kg}$ $v_{bi} = 0 \text{ m/s}$

$v_f = 2.42 \text{ m/s}$

There is an inelastic collision ... and no Net external forces

$$m_k v_{ki} + m_b v_{bi} = (m_k + m_b) v_f$$

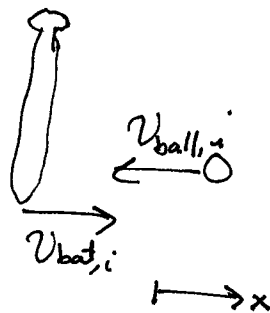
$$v_{ki} = \frac{(m_k + m_b) v_f - m_b v_{bi}}{m_k} \rightarrow 0$$

$$v_{ki} = v_f \left(1 + \frac{m_b}{m_k} \right) = 3.02 \text{ m/s}$$

⑩ Given: $v_{ball,i} = 35 \text{ m/s}$

$$v_{bat,i} = 23 \text{ m/s}$$

$$M_{bat} \gg m_{ball}$$



There is an elastic collision in 1D

let ball = object 1

$$v_{1i} = -v_{ball}$$

bat = object 2

$$v_{2i} = v_{bat}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

but since $m_2 \gg m_1$,

$$m_1 - m_2 \approx -m_2$$

$$m_1 + m_2 \approx m_2$$

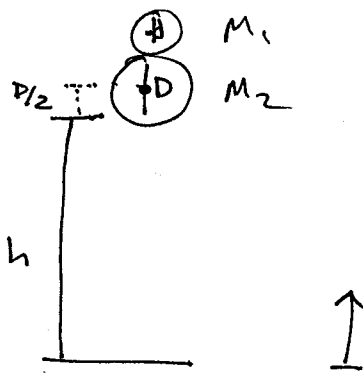
$$v_{1f} = \frac{-m_2}{m_2} (-v_{ball}) + \frac{2m_2}{m_2} (v_{bat})$$

$$v_{1f} = v_{ball} + 2v_{bat} = 81 \text{ m/s}$$

⑪ factor increase = $\frac{KE_f}{KE_i} = \frac{\frac{1}{2} m_{ball} (v_{ball} + 2v_{bat})^2}{\frac{1}{2} m_{ball} (v_{ball})^2}$

$$\text{factor} = \frac{(v_{ball} + 2v_{bat})^2}{(v_{ball})^2} = 5.36$$

⑫ Given: $h = 7.0 \text{ m}$
 $D = 0 \text{ cm}$
 $d = 0 \text{ cm}$
 $M_1 = 0.12 \text{ kg}$
 $M_2 = 1.80 \text{ kg}$



Ball 2 drops to ground

$$E_i + \Sigma W = E_f$$

$$\frac{1}{2}m(0)^2 + mg(h + \frac{D}{2}) + 0 = \frac{1}{2}mV_{2,d}^2 + mg(\frac{D}{2})$$

$$V_{2,d} = \sqrt{2gh}$$

Then ball 2 collides elastically w/ the ground. Since Mechanical energy is conserved $V_{2,o} = V_{2,d}$ but it is now going up.

At the same time, ball 1 drops down

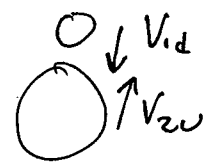
$$E_i + \Sigma W = E_f$$

$$\frac{1}{2}M_1(0)^2 + M_1g(h + D + \frac{d}{2}) + 0 = \frac{1}{2}M_1V_{1,d}^2 + M_1g(D + \frac{d}{2})$$

$$V_{1,d} = \sqrt{2gh}$$

now, M_1 and M_2 collide elastically

$$V_{1,f} = \frac{M_1 - M_2}{M_1 + M_2} V_{1,i} + \frac{2M_2}{M_1 + M_2} V_{2,i}$$



$$V_{1,f} = \frac{M_1 - M_2}{M_1 + M_2} (\sqrt{2gh}) + \frac{2M_2}{M_1 + M_2} (\sqrt{2gh})$$

$$V_{1,f} = \sqrt{2gh} \left(\frac{2M_2 + M_2 - M_1}{M_1 + M_2} \right) = \sqrt{2gh} \left(\frac{3M_2 - M_1}{M_1 + M_2} \right)$$

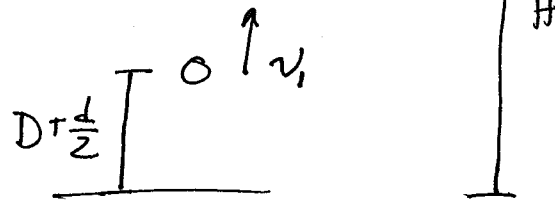
finally we can use energy conservation to find the final height of ball 1

$$E_i + \Sigma W = E_f$$

$$\frac{1}{2} m v_{if}^2 + mg(D + \frac{d}{2}) = mgH$$

$$\frac{1}{2} m_1 (2gh) \left(\frac{3m_2 - m_1}{m_1 + m_2} \right)^2 + M_1 g \left(D + \frac{d}{2} \right) = M_1 g H$$

$$H = D + \frac{d}{2} + h \left(\frac{3m_2 - m_1}{m_1 + m_2} \right)^2 = 52.9 \text{ m}$$



13

Given: $m_1 = 40 \text{ kg}$

$L = 1.2 \text{ m}$

$m_{2A} = 50 \text{ kg}$

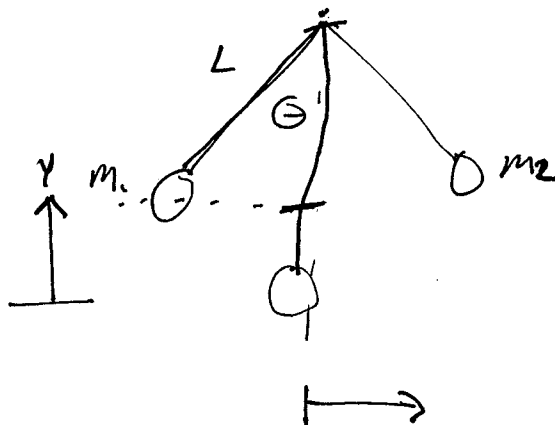
$\theta = 33^\circ$

$B = 25 \text{ kg}$

$C = 40 \text{ kg}$

$D = 70 \text{ kg}$

$E = 15 \text{ kg}$



find v_{zf} (after collision)

$$v_{zf} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

use energy conservation

$$y_i = L - L \cos \theta \quad y_f = 0$$

$$v_i = 0$$

$$v_f = ?$$

$$\Sigma W = 0$$

because Tension is always \perp to

$$\frac{1}{2} m (0)^2 + mg(L - L \cos \theta) = \frac{1}{2} m v_f^2 + mg(0) \quad \text{the motion}$$

$$v_f = \sqrt{2gL(1 - \cos \theta)}$$

$$\text{Clearly } v_{1i} = +v_f \quad \text{and } v_{2i} = -v_f$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} (\sqrt{2gL(1-\cos\theta)}) + \frac{m_2 - m_1}{m_1 + m_2} (-\sqrt{2gL(1-\cos\theta)})$$

$$= \sqrt{2gL(1-\cos\theta)} \left(\frac{2m_1 + m_2 - m_1}{m_1 + m_2} \right)$$

$$V_{2f} = \sqrt{2gL(1-\cos\theta)} \left(\frac{3m_1 - m_2}{m_1 + m_2} \right)$$

so... we need to rank $\left(\frac{3m_1 - m_2}{m_1 + m_2} \right)$ for the five cases...

$$A = 0.41$$

$$B = 0.65$$

$$C = 0.5$$

$$D = 0.263$$

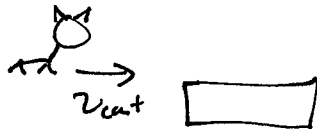
$$E = 0.777$$

so $D < A < C < B < E$

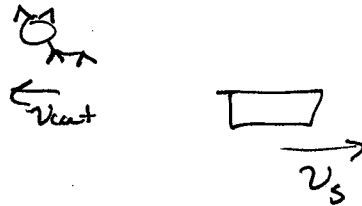
⑭ Given: $m_{\text{sled}} = 19.0 \text{ kg}$
 $m_{\text{cat}} = 4.4 \text{ kg}$
 $v_{\text{cat}} = 2.22 \text{ m/s}$

The cat "collides" with sled 2

initial



final



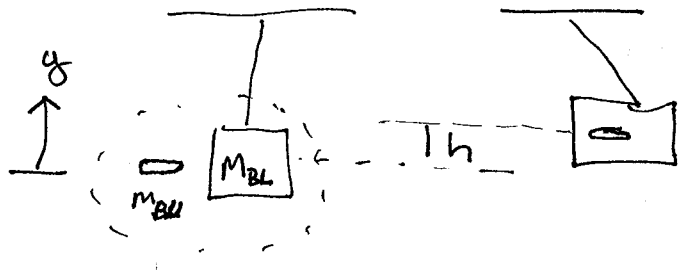
$$P_{ix} + \int F_x dt = P_{fx}$$

$$m_c v_c + 0 = m_c (-v_c) + m_s v_s$$

$$\boxed{v_s = \frac{2m_c v_c}{m_s}} = (1.03 \text{ m/s})$$

15

Given: $m_{bu} = 8.1g = 8.1 \times 10^{-3}g$
 $m_{bl} = 3.2kg$
 $h = 21cm = 0.21m$



First, let's examine the collision.

$$P_{ix} + \cancel{\Sigma F_x \Delta t} = P_{fx}$$

$$m_{bu} v_{bu} + 0 = (m_{bu} + m_{bl}) v$$

↑
the velocity of block and bullet immediately after impact

Now let's examine what happens after the collision... it's an energy conservation problem

$$E_i + \Sigma W = E_f$$

initial
 $v_i = v$
 $y_i = 0$

final
 $v_f = 0$
 $y_f = h$

$$\frac{1}{2} (\cancel{m_{bu} + m_{bl}}) v^2 = (\cancel{m_{bu} + m_{bl}}) gh$$

$$v = \sqrt{2gh}$$

plug into momentum conservation

$$m_{bu} v_{bu} = (m_{bu} + m_{bl}) v$$

$$v_{bu} = \left(\frac{m_{bu} + m_{bl}}{m_{bu}} \right) \sqrt{2gh}$$

$$= 803.5 \text{ m/s}$$

The only force other than gravity acting on the block/bullet is Tension. T does no work here because it is perpendicular to the motion
 $\Sigma W = 0$

16 Given

$$m_b = 6.60 \text{ g} = 6.6 \times 10^{-3} \text{ kg}$$

$$m_1 = 1.70 \text{ kg}$$

$$m_2 = 1.65 \text{ kg}$$

$$v_1 = 3.40 \text{ m/s}$$

$$v_2 = 1.80 \text{ m/s}$$

an inelastic collision!

initial



$$P_{ix} + \cancel{\sum F_x \Delta t} = P_{fx}$$

$$m_b v_b + 0 = m_1 v_1 + m_2 v_2 + m_b v_2$$

$$v_b = \frac{m_1 v_1 + m_2 v_2 + m_b v_2}{m_b}$$

$$v_b = v_2 + \frac{m_1 v_1 + m_2 v_2}{m_b} = (1.33 \times 10^3 \text{ m/s})$$

(17)

Given: $W_A = 1020 \text{ N}$

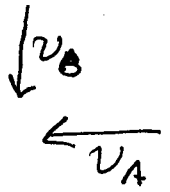
$V_A = 46 \text{ m/s}$

$W_B = 1750 \text{ N}$

$V_B = 25 \text{ m/s}$

$M_A = \frac{W_A}{g}$

$M_B = \frac{W_B}{g}$

This is a 2D elastic collision

$P_{ix} + \sum F_x \Delta t = P_{fx}$

$P_{iy} + \sum F_y \Delta t = P_{fy}$

$m_A(v_A) + 0 = P_{fx} = (M_A + M_B)v_{fx}$

$M_B(-v_B) + 0 = P_{fy} = (M_A + M_B)v_{fy}$

$$v_{fx} = \frac{-M_A v_A}{M_A + M_B} = \frac{-\frac{W}{g}}{\frac{W_A}{g} + \frac{W_B}{g}} v_A$$

$$v_{fy} = \frac{-M_B v_B}{M_A + M_B}$$

$$v_{fx} = \frac{-W_A}{W_A + W_B} v_A$$

$$v_{fy} = \frac{-W_B}{W_A + W_B} v_B$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$

$$v_f = \frac{1}{W_A + W_B} \sqrt{(W_A v_A)^2 + (W_B v_B)^2} = (2.32 \times 10^1 \text{ m/s})$$

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$$\theta = \tan^{-1} \left(\frac{v_{fy}}{v_{fx}} \right) = \tan^{-1} \left(\frac{\frac{-W_B}{W_A + W_B} v_B}{\frac{-W_A v_A}{W_A + W_B}} \right)$$

$$= \tan^{-1} \left(\frac{-W_B v_B}{-W_A v_A} \right)$$

Be careful. The answer is in the third quadrant, so your calculator will give you the wrong answer and so you must add 180° .