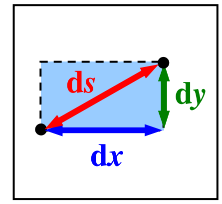
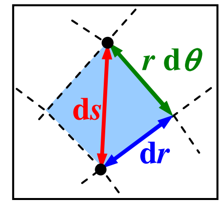
Coordinate System Basics



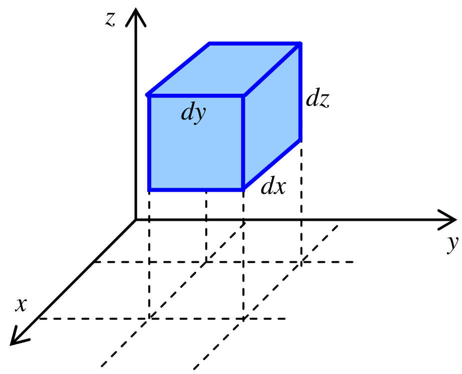
Pythagorean theorem, Cartesian: $ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$



Pythagorean theorem, 2D cylindrical: $ds = \sqrt{dr^2 + (rd\theta)^2} = dr\sqrt{1 + \left(r\frac{d\theta}{dr}\right)^2}$

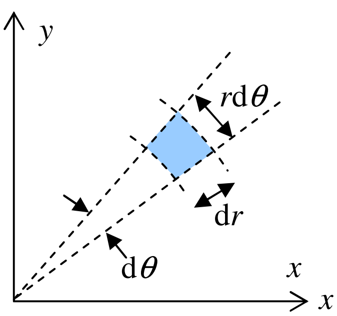
CARTESIAN (page 260)

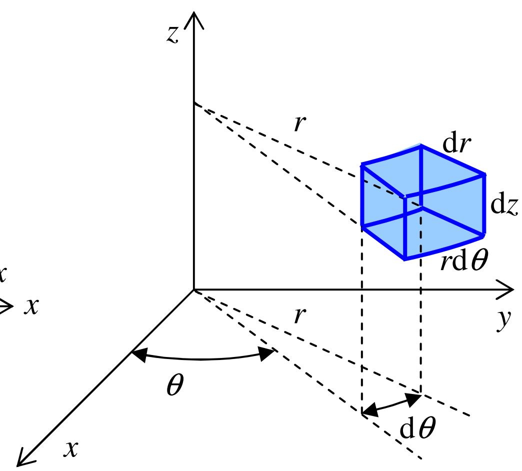
$x = x$
$y = y$
$z = z$
$\mathrm{d}V = \mathrm{d}x \cdot \mathrm{d}y \cdot \mathrm{d}z$
$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$



CYLINDRICAL (page 260)

$x = r \cdot \cos\theta$
$y = r \cdot \sin\theta$
$z = z$
$\mathrm{d}V = \mathrm{d}x \cdot \mathrm{d}y \cdot \mathrm{d}z = r \cdot \mathrm{d}r \cdot \mathrm{d}\theta \cdot \mathrm{d}z$
$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + \mathrm{d}z^2$



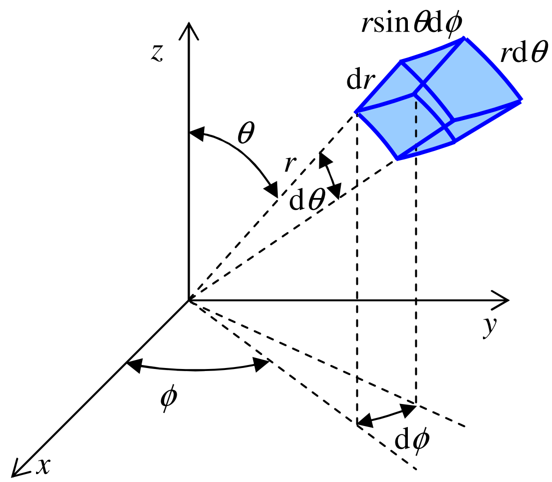


SPHERICAL (page 261)

$x = r \cdot \sin\theta\cos\phi$
$y = r \cdot \sin\theta\sin\phi$
$z = r \cdot \cos\theta$
$\mathrm{d}V = r^2\sin\theta \cdot \mathrm{d}r \cdot \mathrm{d}\theta \cdot \mathrm{d}\phi$
$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta \cdot \mathrm{d}\phi^2$

Vector ds: $d\vec{s} = (dr)\hat{r} + (rd\theta)\hat{\theta} + (r\sin\theta d\phi)\hat{\phi}$

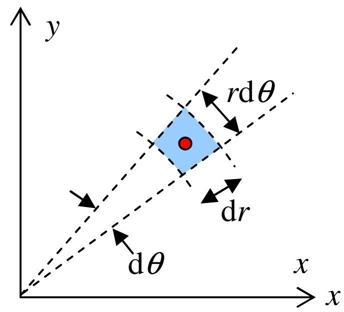
$\vec{V} = \frac{d\vec{s}}{dt} = \frac{\partial r}{\partial t}\hat{r} + r\frac{\partial\theta}{\partial t}\hat{\theta} + r\sin\theta\frac{\partial\phi}{\partial t}\hat{\phi}.$

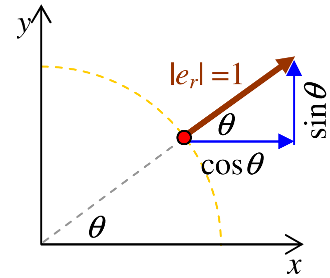


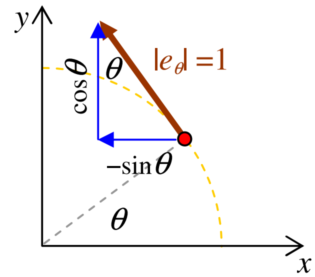
Unit Vectors… 2D Cylindrical:

$\hat{e}_r = \cos\theta\hat{i} + \sin\theta\hat{j}$, and

$\hat{e}_\theta = -\sin\theta\hat{i} + \cos\theta\hat{j}$, and $\hat{e}_z = 1\hat{k}$







Scale Factors

Cartesian coordinates:	$x_1 = x$	$x_2 = y$	$x_3 = z$
Cylindrical coordinates:	$x_1 = r$	$x_2 = \theta$	$x_3 = z$
Spherical coordinates:	$x_1 = r$	$x_2 = \theta$	$x_3 = \phi$

All coordinates: $d\vec{s} = h_1(\partial x_1)\hat{e}_{x_1} + h_2(\partial x_2)\hat{e}_{x_2} + h_3(\partial x_3)\hat{e}_{x_3}$

Cartesian coordinates:	$h_x = 1$,	$h_y = 1$,	$h_z = 1$.
Cylindrical coordinates:	$h_r = 1$,	$h_\theta = r$,	$h_z = 1$.
Spherical coordinates:	$h_r = 1$,	$h_\theta = r$,	$h_\phi = r\sin\theta$.

Using Scale Factors

Gradient: $$\vec{\nabla}U = \frac{1}{h_1}\frac{\partial U}{\partial x_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial U}{\partial x_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial U}{\partial x_3}\hat{e}_3$$

Divergence: $$\vec{\nabla}\cdot\vec{V} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}(h_2h_3V_1) + \frac{\partial}{\partial x_2}(h_3h_1V_2) + \frac{\partial}{\partial x_3}(h_1h_2V_3)\right]$$

Laplacian: $$\vec{\nabla}^2U = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}\left(\frac{h_2h_3}{h_1}\frac{\partial U}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\frac{h_3h_1}{h_2}\frac{\partial U}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(\frac{h_1h_2}{h_3}\frac{\partial U}{\partial x_3}\right)\right]$$

Curl: $$\vec{\nabla}\times\vec{V} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ h_1V_1 & h_2V_2 & h_3V_3 \end{vmatrix}$$