The Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}, \text{ where } \binom{n}{k} = \frac{n!}{k!(n-k)!} = \text{the “choose function”}$$

$n = 0$: $(a + b)^0 = 1a^0b^0 = 1$

$n = 1$: $(a + b)^1 = 1a^1b^0 + 1a^0b^1 = a + b$

$n = 2$: $(a + b)^2 = a^2b^0 + 2a^1b^1 + a^0b^2 = a^2 + 2ab + b^2$

$n = 3$: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$n = 4$: $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Pascal's Triangle

$n = 0$: 1

$n = 1$: 1 1

$n = 2$: 1 2 1

$n = 3$: 1 3 3 1

$n = 4$: 1 4 6 4 1

$n = 5$: 1 5 10 10 5 1

$n = 6$: 1 6 15 20 15 6 1

$n = 7$: 1 7 21 35 35 21 7 1

$n = 8$: 1 8 28 56 70 56 28 8 1

Geometric Series:

$$\sum_{i=0}^{n-1} ar^i = \frac{a\left(1-r^n\right)}{1-r} \qquad \sum_{i=0}^{n} ar^i = \frac{a\left(1-r^{n+1}\right)}{1-r}$$

if $|r| < 1$:

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r} \qquad \sum_{i=1}^{\infty} ar^i = \frac{ar}{1-r}$$

Power Series Expansions

Assuming that $\sin(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$, then solving for a_i and simplifying results in:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots$$

Similarly,

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots$$

Clarification of Equations 1.4 and 1.8 on page 2 of Boas:

$$\sum_{i=0}^{n-1} ar^i = \frac{a\left(1-r^n\right)}{1-r}$$

$$\sum_{i=0}^{n} ar^i = \frac{a\left(1-r^{n+1}\right)}{1-r}$$

$$\sum_{i=1}^{n} ar^i = \sum_{i=0}^{n} ar^i - a$$

$$\left.\begin{aligned} \sum_{i=0}^{\infty} ar^i &= \frac{a}{1-r} \\ \sum_{i=1}^{\infty} ar^i &= \frac{ar}{1-r} \end{aligned}\right\} \quad r^2 < 1$$