

The Binomial Theorem

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}, \text{ where } \binom{n}{k} = \frac{n!}{k!(n-k)!} = \text{the "choose function"}$$

$$n = 0: (a + b)^0 = 1a^0b^0 = 1$$

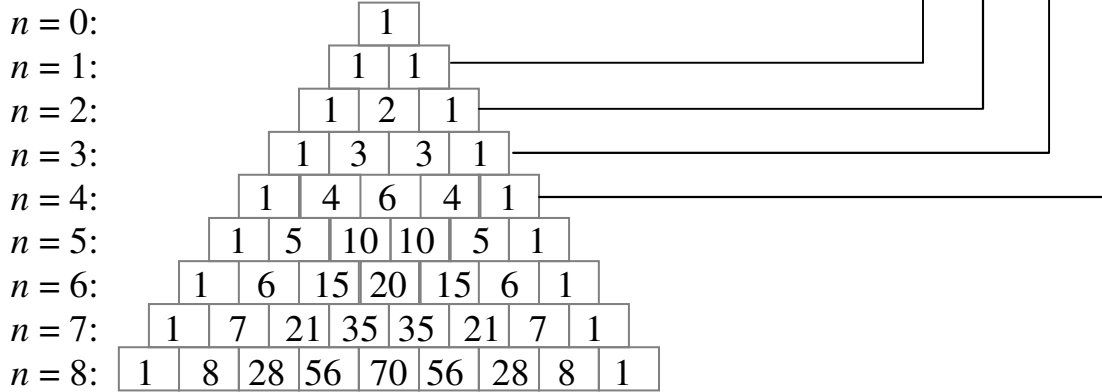
$$n = 1: (a + b)^1 = 1a^1b^0 + 1a^0b^1 = a + b$$

$$n = 2: (a + b)^2 = a^2b^0 + 2a^1b^1 + a^0b^2 = a^2 + 2ab + b^2$$

$$n = 3: (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$n = 4: (a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Pascal's Triangle



Geometric Series:

$$\sum_{i=0}^{n-1} ar^i = \frac{a(1-r^n)}{1-r}$$

$$\sum_{i=0}^n ar^i = \frac{a(1-r^{n+1})}{1-r}$$

if $|r| < 1$

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r} \quad \sum_{i=1}^{\infty} ar^i = \frac{ar}{1-r}$$

Power Series Expansions

Assuming that $\sin(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + \dots$, then solving for a_i and simplifying results in:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

Similarly,

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$

Clarification of Equations 1.4 and 1.8 on page 2 of Boas:

$$\sum_{i=0}^{n-1} ar^i = \frac{a(1-r^n)}{1-r}$$

$$\sum_{i=0}^n ar^i = \frac{a(1-r^{n+1})}{1-r}$$

$$\sum_{i=1}^n ar^i = \sum_{i=0}^n ar^i - a$$

$$\left. \begin{aligned} \sum_{i=0}^{\infty} ar^i &= \frac{a}{1-r} \\ \sum_{i=1}^{\infty} ar^i &= \frac{ar}{1-r} \end{aligned} \right\} r^2 < 1$$