The Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}, \text{ where } \binom{n}{k} = \frac{n!}{k!(n-k)!} = \text{the “choose function”}$$

$n = 0$: $(x + y)^0 = 1x^0y^0 = 1$
$n = 1$: $(x + y)^1 = 1x^1y^0 + 1x^0y^1 = x + y$
$n = 2$: $(x + y)^2 = x^2y^0 + 2x^1y^1 + x^0y^2 = x^2 + 2xy + y^2$
$n = 3$: $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
$n = 4$: $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Pascal's Triangle

$n = 0$:	1
$n = 1$:	1 1
$n = 2$:	1 2 1
$n = 3$:	1 3 3 1
$n = 4$:	1 4 6 4 1
$n = 5$:	1 5 10 10 5 1
$n = 6$:	1 6 15 20 15 6 1
$n = 7$:	1 7 21 35 35 21 7 1
$n = 8$:	1 8 28 56 70 56 28 8 1

Geometric Series:

$$\sum_{i=0}^{n-1} ar^i = \frac{a\left(1-r^n\right)}{1-r} \qquad \sum_{i=0}^{n} ar^i = \frac{a\left(1-r^{n+1}\right)}{1-r}$$

if $|r| < 1$:

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r} \qquad \sum_{i=1}^{\infty} ar^i = \frac{ar}{1-r}$$

Power Series Expansions

Assuming that $\sin(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + \ldots$, then solving for a_i and simplifying results in:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots$$

Similarly,

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots$$