Question: Find the cube root of *i*.
Summary: write *i* in exponential form, then raise it to the one-third power, then convert it back into *x* + *iy* form using sine and cosine.

First, let $i = 1e^{\frac{i\pi}{2}}$; then: $i^{\frac{1}{3}} = \left(1e^{\frac{i\pi}{2}}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}} e^{\frac{i\pi}{6}} = e^{\frac{i\pi}{6}} = \cos\left(\frac{\pi}{6}\right) + \sin(\frac{\pi}{6})i = \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2}i}$

Or, let $i = 1e^{2\pi i + \frac{i\pi}{2}}$; then: $\left(1e^{2\pi i + \frac{i\pi}{2}}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}}\left(e^{\frac{2}{3}\pi i + \frac{i\pi}{6}}\right) = \boxed{e^{\frac{5i\pi}{6}} = \frac{-\sqrt{3}}{2} + \frac{i}{2}}$

Or, let $i = 1e^{4\pi i + \frac{i\pi}{2}}$; then: $\left(1e^{4\pi i + \frac{i\pi}{2}}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}}\left(e^{\frac{4}{3}\pi i + \frac{i\pi}{6}}\right) = e^{\frac{9i\pi}{6}} = \boxed{e^{\frac{3i\pi}{2}} = -i}$

I could also attempt to let $i = 1e^{6\pi i + \frac{i\pi}{2}}$; then: $\left(1e^{6\pi i + \frac{i\pi}{2}}\right)^{\frac{1}{3}} = 1^{\frac{1}{3}}\left(e^{2\pi i + \frac{i\pi}{6}}\right) = e^{\frac{i\pi}{6}} = \frac{\sqrt{3}}{2} + \frac{i}{2}$

Note that this fourth answer is the same as the first, so it isn't useful or "new".

As hoped for, we were able to obtain three *unique* answers for the cube root of *i*.

We could check each answer by raising the result to the third power, and seeing if we get *i* again:

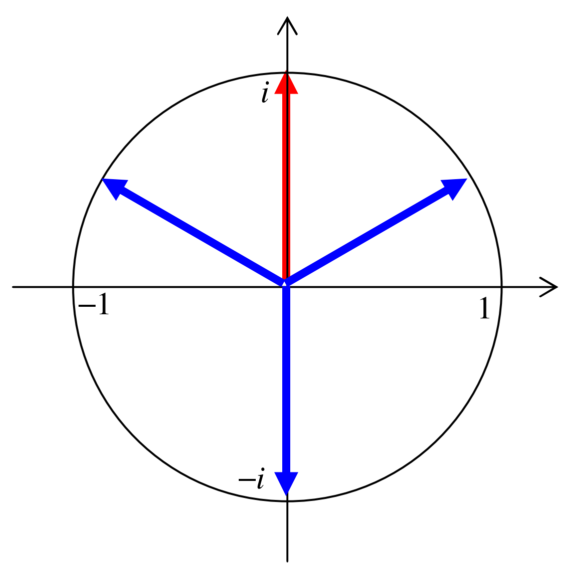
$\left(e^{\frac{i\pi}{6}}\right)^3 = e^{\frac{3i\pi}{6}} = e^{\frac{i\pi}{2}} = i$ Good!

$\left(e^{\frac{5i\pi}{6}}\right)^3 = e^{\frac{15i\pi}{6}} = e^{\frac{5i\pi}{2}} = \left(e^{\frac{4i\pi}{2}}\right)\left(e^{\frac{1i\pi}{2}}\right) = 1 \cdot e^{\frac{1i\pi}{2}} = i$ Good!

$\left(e^{\frac{3i\pi}{2}}\right)^3 = {}^{\frac{9i\pi}{2}} = \left(e^{\frac{8i\pi}{2}}\right)\left(e^{\frac{1i\pi}{2}}\right) = 1 \cdot e^{\frac{1i\pi}{2}} = i$ Good!

When plotted as vectors in the complex plane, these roots all have the same magnitude (just 1.0) and are all evenly spaced with respect to each other (obviously, the angles are $\theta = \pi/6$, $\theta = 5\pi/6$, and $\theta = 3\pi/2$).

You should expect even-spacing for any root problem you solve. Cube roots are all spaced at 120°. For 4th roots, all results will be spaced at 90°. For 5th roots, they will be spaced at 72°, and so on.



The "Complex Plane"