Suppose you measure that temperature is related to pressure and density by: $T = k_1 \dfrac{P}{\rho}$ [1]

Also, you measure pressure and density as functions of time: $P = k_2 t^2$ [2]

and $\rho = k_3 t^4$ [3]

You want to know the rate of change of temperature with respect to time. In this problem, you could do it pretty easily by straight substitution:

Since $T = k_1 \dfrac{P}{\rho} = k_1 \dfrac{k_2 t^2}{k_3 t^4} = \dfrac{k_1 k_2}{k_3} \dfrac{1}{t^2} = \dfrac{k_1 k_2}{k_3} t^{-2}$

Therefore, $\boxed{\dfrac{\partial T}{\partial t} = \dfrac{-2k_1 k_2}{k_3} t^{-3}}$.

However, sometimes, it's easier to use differentials (but not in this problem!):

$\boxed{\dfrac{\partial T}{\partial t} = \dfrac{\partial T}{\partial P}\dfrac{dP}{dt} + \dfrac{\partial T}{\partial \rho}\dfrac{d\rho}{dt}}$, where:

$\dfrac{\partial T}{\partial P} = \dfrac{k_1}{\rho}$ from [1]

$\dfrac{\partial T}{\partial \rho} = \dfrac{-k_1 p}{\rho^2}$ from [1]

$\dfrac{\partial P}{\partial t} = 2k_2 t$ from [2]

$\dfrac{\partial \rho}{\partial t} = 4k_3 t^3$ from [3]

Plugging these 4 derivatives into the differential, we get:

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial p}\frac{dp}{dt} + \frac{\partial T}{\partial \rho}\frac{d\rho}{dt} = \left(\frac{k_1}{\rho}\right)(2k_2 t) + \left(\frac{-k_1 p}{\rho^2}\right)(4k_3 t^3)$$

Substitute for p, ρ from [2], [3]

$$\frac{\partial T}{\partial t} = \left(\frac{k_1}{k_3 t^4}\right)(2k_2 t) + \left(\frac{-k_1 (k_2 t^2)}{(k_3 t^4)^2}\right)(4k_3 t^3)$$

Multiply everything together:

$$\frac{\partial T}{\partial t} = \left(\frac{2k_1 k_2}{k_3 t^3}\right) - \left(\frac{4k_1 k_2}{k_3 t^3}\right) \qquad \boxed{\frac{\partial T}{\partial t} = -\left(\frac{2k_1 k_2}{k_3 t^3}\right)}$$

We get the same answer either way. In some problems the differentials method is easier. In some, the substitution method is easier.