

**Example:** Compute curl of velocity from Monday's class:  $V_x = 6xy + 1.5y^2 - 1.5x^2 + 2$   
 $V_y = 3x^2 + 3xy - 3y^2 + 1$

$$\bar{\nabla} \times \bar{V} = \left( \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) \hat{i} + \left( \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) \hat{j} + \left( \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \hat{k}$$

$$\bar{\nabla} \times \bar{V} = 0\hat{i} + 0\hat{j} + ((6x + 3y) - (6x + 3y))\hat{k} = 0$$

→ We knew this had to be true, since  $\phi$  existed and was already found in class!

**Example:** Given  $V_x = 6xy + y^2$ , what is  $V_y$  if the flow is incompressible?

$$\bar{\nabla} \cdot \bar{V} = 0 = (6y) + \frac{\partial V_y}{\partial y} \rightarrow V_y = \int -6y dy$$

$$V_y = -6 \frac{y^2}{2} + C(x) = -3y^2 + C(x)$$

**Example:** Given

$V_x = 5 - y$ , and  $V_y = x - 5$ . Find curl, divergence,  $\phi$ , and  $\psi$ .

$$\bar{\nabla} \times \bar{V} = 0\hat{i} + 0\hat{j} + (1 - (-1))\hat{k} = -2\hat{k} \frac{1}{s} \quad \text{Not Zero!!}$$

$$\bar{\nabla} \cdot \bar{V} = 0 + 0 = 0 \quad \text{So this flow is incompressible.}$$

$$\psi = 5y - \frac{y^2}{2} + C_1(x) \quad \text{and} \quad \psi = -\frac{x^2}{2} + 5x + C_2(y)$$

$$\therefore C_1(x) = -\frac{x^2}{2} + 5x \quad \text{and} \quad C_2(y) = 5y - \frac{y^2}{2}$$

$$\psi = 5y - \frac{y^2}{2} - \frac{x^2}{2} + 5x \quad \text{Can also add any constant to this answer.}$$

$$\phi = 5x - xy + C_3(y) \quad \text{and} \quad \phi = xy - 5y + C_4(x) ???$$

$\phi$  doesn't exist!

"Solid Body rotation"... this is a rotating disk. Outer points move faster.

**Example:** Given:  $\bar{V} = (x^2 - y^2)\hat{i} + (-2xy)\hat{j}$ . Find  $\phi$  and  $\psi$ .

$$\psi = x^2y - \frac{y^3}{3} + C_1(x) \quad \text{and} \quad \psi = +2\frac{x^2}{2}y + C_2(y)$$

$$\therefore C_1(x) = 0 \quad \text{and} \quad C_2(y) = -\frac{y^3}{3}$$

$$\psi = x^2y - \frac{y^3}{3}$$

Can also add any constant to this answer.

$$\phi = \frac{x^3}{3} - y^2x + C_1(y) \quad \text{and} \quad \phi = -2x\frac{y^2}{2} + C_2(x) \rightarrow C_1(y) = 0 \quad \text{and} \quad C_2(x) = \frac{x^3}{3}$$

$$\phi = \frac{x^3}{3} - y^2x$$

Can also add any constant to this answer.