Problem 1

$$x^4 + 62\cdot x^2 + 288\cdot x + 1825 \text{ solve } \rightarrow \begin{pmatrix} -3+4i \\ -3-4i \\ 3+8i \\ 3-8i \end{pmatrix}$$

The answer asked for was the second of these:

-3 - 4*i

Problem 2

$\mu := 7000 \qquad \sigma := 500$

$\text{pnorm}(7500,\mu,\sigma) - \text{pnorm}(7500,\mu,\sigma) = 0$

zero

Problem 3

$\text{pnorm}(7600,\mu,\sigma) - \text{pnorm}(6600,\mu,\sigma) = 0.673$

0.673

Problem 4 We are given four equations in sentence form. Let's convert them into math notation:

$A = \dfrac{A+B+C+D}{4}$ In other words, $3\cdot A - 1B - 1C - 1D = 0$

$B = \dfrac{A+C+D}{3} - 12$ $\qquad -1A + 3\cdot B - 1C - 1D = -36$

$C = \dfrac{B+D}{2} + 9$ $\qquad 0\cdot A - 1B + 2\cdot C - 1D = 18$

$D = 10\cdot(A-B) - 4$ $\qquad 10\cdot A - 10\cdot B + 0\cdot C - 1D = 4$

In Matrix notation, this becomes:

$$M := \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 10 & -10 & 0 & -1 \end{pmatrix} \qquad K := \begin{pmatrix} 0 \\ -36 \\ 18 \\ 4 \end{pmatrix}$$

Therefore, the exam scores are:

$\text{Grades} := M^{-1}\cdot K$

$$\text{Grades} = \begin{pmatrix} 83 \\ 74 \\ 89 \\ 86 \end{pmatrix}$$

So, it looks like Bobby got a 74 on this test.

Problem 5

Also, the highest grade was Cassie (89) on this test.

Problem 6

$$\chi(k) := k\cdot\begin{pmatrix} -7 \\ 9i \end{pmatrix} \qquad \chi\text{star}(k) := k\cdot\begin{pmatrix} -7 \\ -9i \end{pmatrix} \qquad \chi\text{star}(k)^T\cdot\chi(k) - 1 \text{ solve } \rightarrow \begin{pmatrix} \dfrac{\sqrt{130}}{130} \\ -\dfrac{\sqrt{130}}{130} \end{pmatrix}$$

We probably want the positive answer.... $k := \dfrac{\sqrt{130}}{130}$

$k = 0.0877$

Problem 7

$\text{time} := \begin{pmatrix} 3 \\ 6 \\ 9 \\ 12 \\ 15 \end{pmatrix}$ $\quad \text{pos} := \begin{pmatrix} 50 \\ 63 \\ 63 \\ 41 \\ 6 \end{pmatrix}$

$$M := \begin{bmatrix} \sum_{i=0}^{4} (\text{time}_i)^0 & \sum_{i=0}^{4} (\text{time}_i)^1 & \sum_{i=0}^{4} (\text{time}_i)^2 \\ \sum_{i=0}^{4} (\text{time}_i)^1 & \sum_{i=0}^{4} (\text{time}_i)^2 & \sum_{i=0}^{4} (\text{time}_i)^3 \\ \sum_{i=0}^{4} (\text{time}_i)^2 & \sum_{i=0}^{4} (\text{time}_i)^3 & \sum_{i=0}^{4} (\text{time}_i)^4 \end{bmatrix} \qquad k := \begin{bmatrix} \sum_{i=0}^{4} \left[(\text{time}_i)^0 \cdot \text{pos}_i\right] \\ \sum_{i=0}^{4} \left[(\text{time}_i)^1 \cdot \text{pos}_i\right] \\ \sum_{i=0}^{4} \left[(\text{time}_i)^2 \cdot \text{pos}_i\right] \end{bmatrix}$$

$$M = \begin{pmatrix} 5 & 45 & 495 \\ 45 & 495 & 6075 \\ 495 & 6075 & 79299 \end{pmatrix} \qquad k = \begin{pmatrix} 223 \\ 1677 \\ 15075 \end{pmatrix}$$

Note that I had to ask MathCAD to show me the answers as decimals instead of in scientific notation.

So it seems that $M_{row3,col1}$ is 495.

And it seems that k_{row2} is 1677.

Problem 8

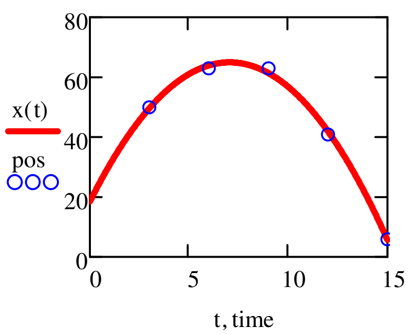
Problem 9

The fit coefficients are: $a := M^{-1} \cdot k$ In other words: $x(t) := a_0 + a_1 \cdot t + a_2 \cdot t^2$

$x(14) = 19.711$

Problem 10

We could plot this to see if this result is reasonable:



Since $x(t) = x_0 + v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$, the acceleration must be:

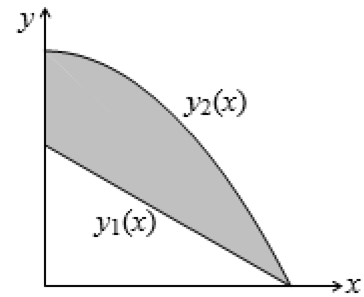
$\frac{1}{2} \cdot a = a_2$, or $a = 2\, a_2$ .In other words:

$a := 2 \cdot a_2$ $\qquad a = -1.873 \ \frac{m}{s^2}$

Problem 11 $\quad \left| \begin{pmatrix} 2 & 3 & -1 \\ 1 & 2 & 0 \\ 4 & 3 & A1 \end{pmatrix} \right| \ \text{solve} \ \rightarrow -5$

Apparently, if A is -5 this problem can't be solved.

Problem 12 $y2(x) := 36 - 4 \cdot x^2$



On the x-axis, when y= 0, $xmax := \sqrt{\frac{-36}{-4}}$

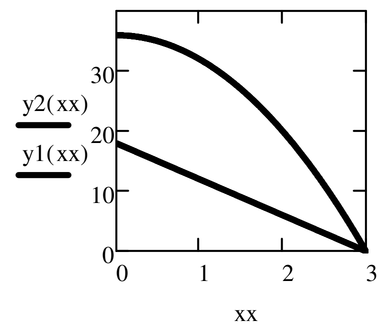
For the straight line, slope = rise/run: $m := \frac{-18}{xmax}$

So, for the straight line: $y1(x) := m \cdot x + 18$

$A := \int_0^{xmax} y2(x) - y1(x)\, dx$ $\quad$ $A = 45$

Problem 13

$$ycm := \frac{\int_0^{xmax} (y2(x) - y1(x)) \cdot \left(\frac{y1(x) + y2(x)}{2}\right) dx}{A}$$ $\quad$ $ycm = 19.44$



Problem 14

$$M := \begin{pmatrix} 0.62932 & 0 & 0.77715 \\ 0 & 1 & 0 \\ -0.77715 & 0 & 0.62932 \end{pmatrix}$$

Note that $|M| = 1$, so this really is a rotation matrix.

Clearly, the middle column is 010, so this matrix rotates about the y-axis. y

There are several ways to find the angle of rotation. We can recall that:

Problem 15

$$Mroty := \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

However, the problem with solving for θ is that there is more than one angle θ such that cos(θ) = 0.62932.Let's find them:

$$\cos(\theta) - 0.62932 \text{ solve } \rightarrow \begin{pmatrix} -0.89011842170412464998 \\ 0.89011842170412464998 \end{pmatrix}$$

Suppose that the correct angle is +0.89011 radians. Then we better hope that +sin(θ) = -0.77715.

$\sin(0.89011) = 0.777$ Hmmm... that isn't right. Let's try the other angle:

$\sin(-0.89011) = -0.777$ That looks better. Let's convert it to degrees:

$-0.890118 \cdot \frac{180}{\pi} = -51$ So, the answer is -51 degrees. However, we were asked to choose an angle between 0 and 358 degrees.

$\theta := -51 + 360$ The answer is therefore 309 deg

An alternative method of solution would be to just have M act on a known vector, such as (1,0,0):

$$M \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.62932 \\ 0.00000 \\ -0.77715 \end{pmatrix} \qquad \theta := \operatorname{atan}\left(\frac{-0.77715}{0.62932}\right)$$

Problem 16

$M := \begin{pmatrix} 6 & 5 \\ 5 & -3 \end{pmatrix}$ $\quad \text{eigenvals}(M) = \begin{pmatrix} -5.227 \\ 8.227 \end{pmatrix}$

Problem 17

$\text{eigenvecs}(M) = \begin{pmatrix} 0.407 & -0.914 \\ -0.914 & -0.407 \end{pmatrix}$

Unfortunately, these aren't formatted the same way that the question asked for. Specifically, the x-component of the 2nd eigenvector should be positive:

$\text{Eigenvectors} := \begin{pmatrix} 0.407 & 0.914 \\ -0.914 & 0.407 \end{pmatrix}$

In the order that the question asked for, this is:

y-component 1: - 0.914
x-component 2: +0.914
y-component 2: +0.407

Problem 18

$m := \begin{pmatrix} 5 \\ 7 \\ 5 \end{pmatrix} \quad x := \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} \quad y := \begin{pmatrix} 0 \\ -2 \\ -3 \end{pmatrix} \quad z := \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$

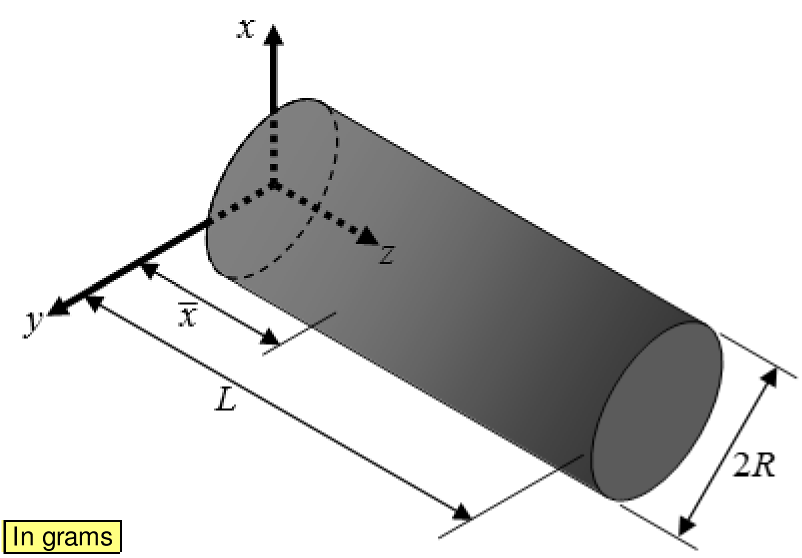
$$Ixy := \sum_{i=0}^{2} \left(-m_i \cdot x_i \cdot y_i\right)$$

Problem 19

$$Iyy := \sum_{i=0}^{2} \left[m_i \cdot \left[\left(x_i\right)^2 + \left(z_i\right)^2\right]\right]$$

Problem 20

$$Izx := \sum_{i=0}^{2} \left(-m_i \cdot x_i \cdot z_i\right)$$



Problem 21

$L := 90 \qquad R := 27$

$\rho(z) := 4 + 23 \cdot \frac{z}{L}$

Check : $\quad \rho(0) = 4 \qquad \rho(L) = 27$

$m := \int_0^L \rho(z) \cdot \pi \cdot R^2 \, dz$ $\quad m = 3.195 \times 10^6$ In grams

Problem 22

$zcm := \dfrac{\int_0^L z \cdot \rho(z) \cdot \pi \cdot R^2 \, dz}{m}$ $\quad zcm = 56.129$ In cm

Problem 23

$Ixx := \int_0^L \int_0^R \int_0^{2\cdot\pi} \rho(z) \cdot r \cdot \left[(r \cdot \sin(\theta))^2 + z^2\right] d\theta \, dr \, dz$ $\quad Ixx = 1.241 \times 10^{10}$ In g*cm^2