zzPogo, E

p228s10 **Section 4**
Due Thr, Feb 11, 2010 at 13:00 Set 3
Physics 228: Math Methods CAPA ID 3479

1. [5pt] The radius of a certain star is currently 1.50×10^{10} m, but it is expanding at a rate of 120 m/s. What is the volumetric rate of expansion (dV/dt)?

2. [5pt] On exam 1, you will be asked to some problems similar to this one without a calculator. You are permitted to use only basic algebra, the derivative of polynomials ($d(ax^n)/dt = nax^{n-1}$), and the chain rule. You may not use the product rule, etc. Find dy/dx at (x = 2.5) if

$$y = \left(\frac{x-2}{x+1}\right)^6$$

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3. [5pt] On exam 1, you will be asked to some problems similar to this one without a calculator. You are permitted to use only basic algebra, the derivative of polynomials ($d(ax^n)/dt = nax^{n-1}$), and the chain rule. You may not use the product rule, etc. Find dy/dx at (x = 3.5) if

$$y = \left(\frac{6x+1}{5x+8}\right)^4$$

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4. [6pt] Reminder: this problem is supposed to be giving you practice with using the chain rule. Your calculator needs no practice. A commonly used approximation of the turbulent velocity u in a pipe carrying any fluid is:

$$u(r) = U_0\left(1 - \frac{r}{R}\right)^{1/7},$$

where R is the physical radius of the pipe, r is any radial position inside the pipe, and U_o is the maximum speed (which happens to occur at the centerline of the pipe at r = 0). The shear stress (τ) is defined by $\tau = \mu(du/dr)$, where μ is called the 'viscosity' of the fluid. Find an expression for $\tau(r)$ in terms of r, R, μ, and U_o. To test your result, determine the value of the shear stress given R = 3 cm, r = 0.90 cm, U_o = 4 m/s, and $\mu = 1.80 \times 10^{-3}$ Ns/m^2.

5. [8pt] Hey! Put away your calculator! Like the other problems, I want you to solve this without any technology higher than a pencil! You still have the restrictions of problem 2!

A researcher studying the flow of the previous problem determines that his uncertainty in r is 0.04 cm, and his uncertainty in R is 0.06 cm. What is the uncertainty in his calculation of u obtained in the prior problem?

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6. [9pt] Given s = 0.8 and n = 5, determine dr/ds, given that

$$r(s) = e^{-p(s)^n - q(s)^n}$$

where $p(s) = e^s$, and $q(s) = e^{-s}$. Like other problems on this assignment, this problem is practice for the chain rule. Start by finding dp/ds and dq/ds.

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7. [6pt] A rectangular steel plate, heated at the edges, is observed to have a temperature profile of $T(x,y) = 12x^4 + 2y^4 - 4xy^2$. There is no heat flow at any point where both $\partial T/\partial x = 0$ and $\partial T/\partial y = 0$. Locate: a) the x-coordinate, and b) the y-coordinate where there is no heat flow.

8. [6pt] The temperature at any point will be steady (that is, not changing over time) if both a) $\partial^2 T/\partial x^2 = 0$ and b) $\partial^2 T/\partial y^2 = 0$. Determine the value of each of these expressions at the same coordinates you found in the previous problem.

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9. [6pt] Given $x = 8.0 \times 10^{31}$, determine the approximate value of

$$\left(\frac{1}{x+1}\right)^6 - \left(\frac{1}{x}\right)^6$$

The next few exercises are to be done on MathCAD. You must electronically submit your one final MathCAD document to my inbox (\\files\Inbox\Physics\Pogo\MathMethods). Emailed versions will be automatically deleted. The name of your file will be 'HW03-LastnameInitial.xmcd'. Use of an incorrect filename (including the use of '.xmcd' twice) is an automatic 10 point deduction. For example, my filename would be 'HW03PogoE.xmcd'. In addition, your name must appear in the document itself.

You can reach the inboxes from any campus computer, or from your personal computer if you have ever bothered to set it up so that it can access the Geneseo domain. Note that accessing the domain is not even remotely related to being able to use the internet while at Geneseo. If your computer does not require you to logon with your Geneseo password every time you turn it on, you are probably not connected to the Geneseo domain.

10. [5] Plot f1(x) between x = -10 and x = +10, where $f1(x) = +x^2$ when x is positive, f1(x) = -50 when x is less than -2, and f1(x) = 0 when x is between -2 and 0. Hint: use the "Add line" feature on the Programming Toolbar.

11. [6] Plot f2(x) between x = -10 and x = +10, where $f2(x) = +x^2$ when x is positive, $f2(x) = -x^2$ when x is negative. Do this problem twice: the first time with an 'if' (as you did in problem 10), and the second time without any conditional expressions. Hint: consider the 'absolute value' function.

12. [5] Plot f2(x,y) for the range of (x,y) between (-5, -5) and (+5, +5), where $f3(x, y) = y^2$ when x is greater than y, and f3(x, y) = 10*sin(x) when x is less than y.

13. [8] Make a 'parametric plot' for the function $A1 = r^3\sin(\theta) - r^3\cos(\theta)$ for values of r between 2 and 8, and values of θ between 0 and 2π.

14. [15] Consider the potential $\phi = (-x)/(x^2 + y^2)$. In the range (x, y) between (1, 1) and (6, 6), plot the potential as a colored field plot with black dividing lines, the streamfunction ψ (dashed black lines without colors), and the vector field arrows, all visible on a single plot.

Images of each desired output are present on the CAPA web version.

Assign #3

① Given $\frac{dr}{dt} = \dot{r}, r \rightarrow$ find $\frac{dV}{dt}$

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = \boxed{\frac{dV}{dt} = 4\pi r^2 \cdot \dot{r}}$

② $y = \left(\frac{x-2}{x+1}\right)^6$ let $v = \frac{x-1}{x+1}$ and $w = x+1$

Note: $x = w-1$, so $v = \frac{(w-1)-2}{w} = \frac{w-3}{w} = 1 - 3w^{-1} = v$

$\frac{dv}{dw} = +3w^{-2} = \frac{3}{(x+1)^2}$

$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx} = (6v^5)\left(\frac{3}{(x+1)^2}\right)(1)$

$$\boxed{\frac{dy}{dx} = 18\frac{(x-2)^5}{(x+1)^7}}$$

(3) $y = \left(\frac{6x+1}{5x+8}\right)^4$ let $v = \frac{6x+1}{5x+8}$ $w = 5x+8$

Note $x = \left(\frac{1}{5}\right)(w-8)$ so $v = \frac{\left(\frac{6}{5}\right)(w-8)+1}{w} = \frac{6}{5} - \frac{43}{5w} = \frac{6}{5} - \frac{43}{5}w^{-1}$

$\frac{dv}{dw} = +\frac{43}{5w^2} = \frac{43}{(5)(5x+8)^2}$

and $\frac{dw}{dx} = 5$

So, $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx} = \left(4v^3\right)\left(\frac{43}{5}\right)\left(\frac{1}{(5x+8)^2}\right)(5)$

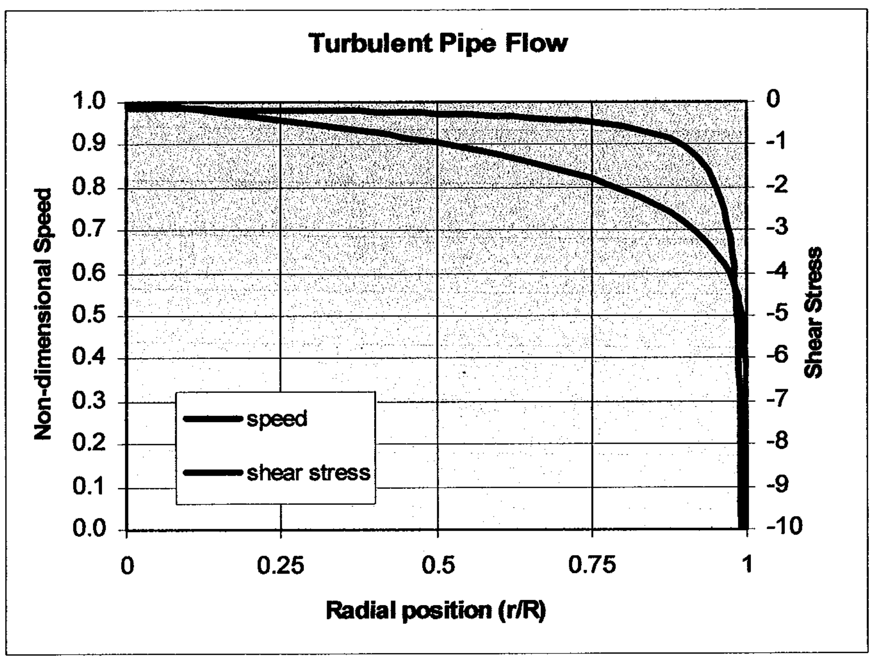
$$\boxed{\frac{dy}{dx} = \frac{(172)(6x+1)^3}{(5x+8)^5}}$$

③ $u(r) = U_0\left(1-\frac{r}{R}\right)^{1/7}$ let $w = 1-\frac{r}{R}$, so $u = w^{1/7}$

$$\frac{du}{dr} = \frac{du}{dw}\cdot\frac{dw}{dr} = \left(\frac{U_0}{7}w^{-\frac{6}{7}}\right)\left(-\frac{1}{R}\right) = -\frac{U_0}{7R}\left(1-\frac{r}{R}\right)^{-6/7}$$

a) $\rightarrow$ $$\boxed{\tau = \frac{-\mu U_0}{7R}\left(1-\frac{r}{R}\right)^{-6/7}}$$

Using $\mu = 1$, $\frac{u(r)}{U_0}$ and $\tau(r)$ are:



u_{MAX} occurs in the middle of the pipe, at $r=0$
τ_{MAX} occurs at the edge of the pipe, at $r=R$

Unfortunately, $\tau(R) \rightarrow -\infty$, which is clearly impossible.

⑤ $$\Delta u^2 = \left(\frac{du}{dr}\Delta r\right)^2 + \left(\frac{\partial u}{\partial R}\Delta R\right)^2$$

We've already done $\frac{du}{dr}$.

CONTINUED

3b, continued: Next, we want $\frac{\partial u}{\partial R}$

As before, let $u = v_0 w^{1/7}$ where $w = 1 - \frac{r}{R}$

and $\frac{dw}{dR} = \frac{+r}{R^2}$

So, $\frac{\partial u}{\partial R} = \frac{\partial u}{\partial w} \cdot \frac{\partial w}{\partial R} = \frac{v_0}{7} w^{-\frac{6}{7}} \cdot \frac{r}{R^2} = \frac{r}{7R^2} \frac{v_0}{\left(1-\frac{r}{R}\right)^{6/7}}$

Putting it all together

$$\boxed{\Delta u = \sqrt{\left(\frac{v_0 \Delta r}{7R\left(1-\frac{r}{R}\right)^{\frac{6}{7}}}\right)^2 + \left(\frac{r}{7R^2} \frac{v_0 \Delta R}{\left(1-\frac{r}{R}\right)^{6/7}}\right)^2}}$$

⑥ $r = e^{-p^n - q^n}$ where $p = e^s$ and $q = e^{-s}$

$\frac{dp}{ds} = e^s$, and $\frac{dq}{ds} = -e^{-s}$

$\frac{dr}{ds} = \frac{\partial r}{\partial p}\frac{dp}{ds} + \frac{\partial r}{\partial q}\cdot\frac{dq}{ds}$

let $w = -p^n - q^n$ so $r = e^w$

then $\frac{dw}{dp} = -np^{n-1}$

and $\frac{dw}{dq} = -nq^{n-1}$

$\frac{\partial r}{\partial p} = \frac{dr}{dw}\cdot\frac{\partial w}{\partial p} = (e^w)(-np^{n-1})$

Similarly, $\frac{\partial r}{\partial q} = \frac{dr}{dw}\cdot\frac{\partial w}{\partial q} = (e^w)(-nq^{n-1})$

Combining all: $\frac{dr}{ds} = (e^w)(-np^{n-1})e^s + (e^w)(-nq^{n-1})(-e^{-s})$

$$\frac{dr}{ds} = (-n)e^w\left[e^s\cdot(e^s)^{n-1} - (e^{-s})(e^{-s})^{n-1}\right]$$

$$\frac{dr}{ds} = (-n)(e^w)\left[e^{ns} - e^{-ns}\right]$$

$$\boxed{\begin{aligned}\frac{dr}{ds} &= (-n)\left(e^{-e^{ns} - e^{-ns}}\right)\left(e^{ns} - e^{-ns}\right)\\ &= -2n\cdot\sinh(n\cdot s)\left(e^{-e^{ns}}\right)\left(e^{-e^{-ns}}\right)\end{aligned}}$$

(7) $T = 12x^4 + 2y^4 - 4xy^2$

$\frac{\partial T}{\partial x} = 48x^3 - 4y^2$ and $\frac{\partial T}{\partial y} = 8y^3 - 8xy$

$\frac{\partial^2 T}{\partial x^2} = 144x^2$ $\qquad$ $\frac{\partial^2 T}{\partial y^2} = 24y^2 - 8x$

We want $\frac{\partial T}{\partial x} = \frac{\partial T}{\partial y} = 0$, so

$48x^3 - 4y^2 = 0$ and $8y^3 - 8xy = 0$

$12x^3 = y^2 \longleftarrow y^2 = x$

$12x^3 = x$

$12x^2 = 1$

$$\boxed{x = \sqrt{\tfrac{1}{12}} = 0.2887, \text{ and } \therefore\ y = 0.5373}$$

(8) Now, we want $\frac{\partial^2 T}{\partial x^2} = 144x^2 = \boxed{12.0 = \frac{\partial^2 T}{\partial x^2}}$

and $\frac{\partial^2 T}{\partial y^2} = 24y^2 - 8x = \boxed{4.619 = \frac{\partial^2 T}{\partial y^2}}$

Therefore, the tempature at this point must be increasing or decreasing with time.

(9) $x = 8 \times 10^{31}$, and $f(x) = \frac{1}{x^6}$

then $\frac{df}{dx} = \frac{-6}{x^7}$

we want $f(n+1) - f(n)$

Note that $\frac{df}{dx} \approx \frac{\Delta f}{\Delta x}$ if Δx is small

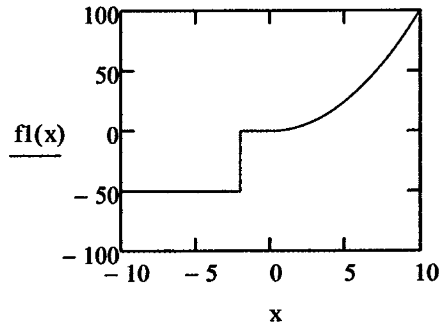
$$\frac{df}{dx} \approx \frac{f(x+1) - f(x)}{(x+1) - (x)} = \boxed{f(x+1) - f(x) = \frac{-6}{x^7}}$$

Our assumption (that Δx is small) requires a large number for x itself. Therefore, we couldn't use this approximation if x was small (for reasonable accuracy, we actually need $x > 400$ or so)

Dr. Pogo's Solutions to Assignment #3

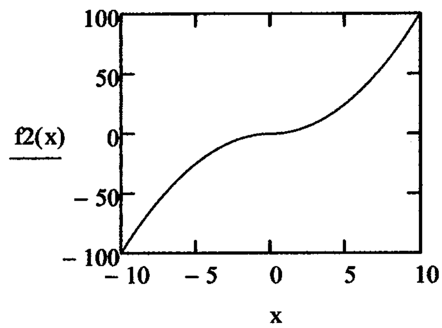
Problem #10

$$f1(x) := \begin{vmatrix} x^2 & \text{if } x > 0 \\ -50 & \text{if } x \le -2 \\ 0 & \text{otherwise} \end{vmatrix}$$



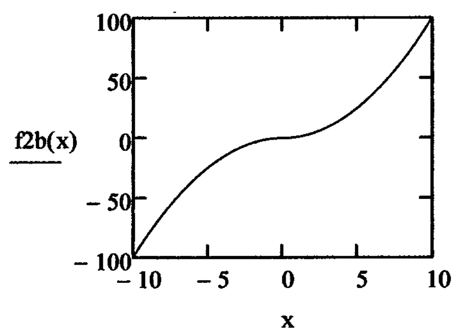
Problem #11

$$f2(x) := \begin{vmatrix} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{otherwise} \end{vmatrix}$$



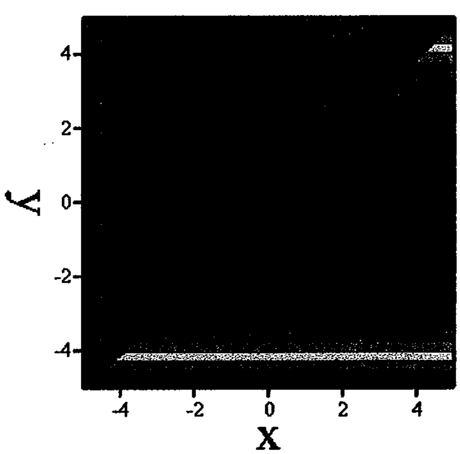
$$f2b(x) := x \cdot |x|$$

Note that $\frac{x^3}{|x|}$ is not defined at the origin.

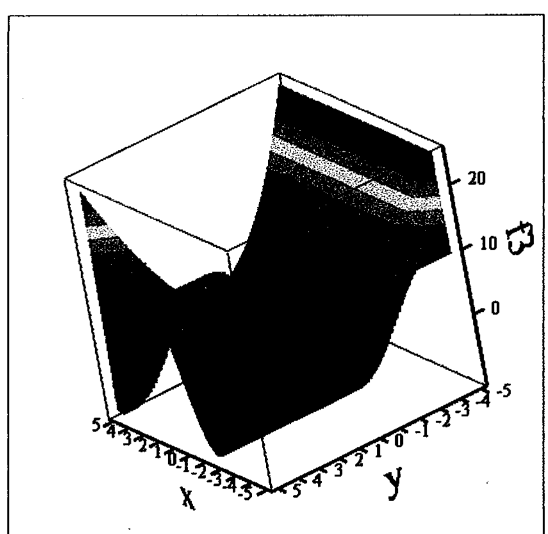


Problem #12

$$f3(x,y) := \begin{vmatrix} y^2 & \text{if } x > y \\ 10 \cdot \sin(x) & \text{otherwise} \end{vmatrix}$$



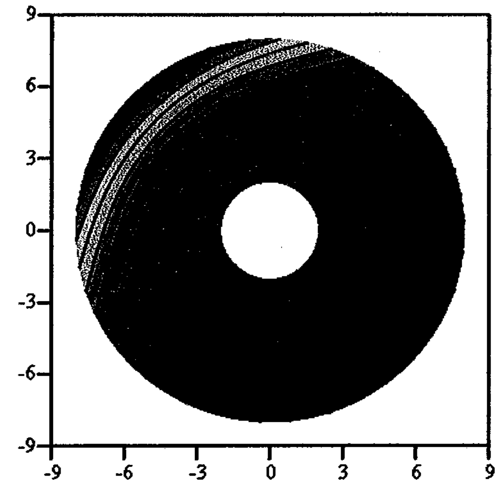
f3



f3

Problem #13 $xx(r,\theta) := r \cdot \cos(\theta)$ $yy(r,\theta) := r \cdot \sin(\theta)$ $A(r,\theta) := r^3 \cdot \sin(\theta) - r^3 \cdot \cos(\theta)$

Note: you have to be careful about calling variable 'x' or 'y', because those names are being used elsewhere in this worksheet.



(xx, yy, A)

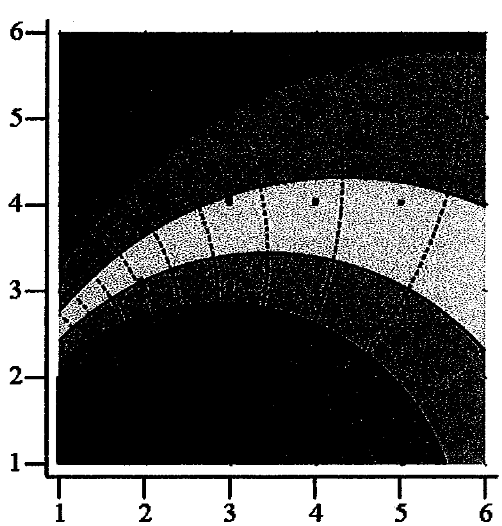
Problem #14 $\phi(x,y) := \dfrac{-x}{x^2 + y^2}$ $v(x,y) := \begin{pmatrix} \dfrac{d}{dx}\phi(x,y) \\ \dfrac{d}{dy}\phi(x,y) \end{pmatrix}$ $\psi1(x,y) := \int v(x,y)_0 \, dy \rightarrow \dfrac{y}{x^2 + y^2}$

$\psi2(x,y) := \int -v(x,y)_1 \, dx \rightarrow \dfrac{y}{x^2 + y^2}$

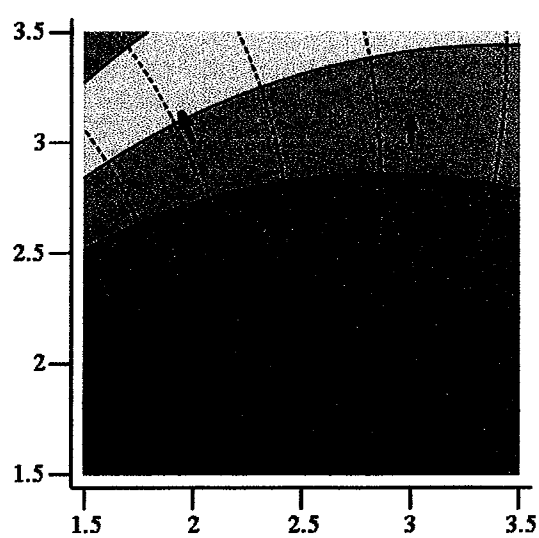
These agree, therefore: $\psi(x,y) := \dfrac{-y}{x^2 + y^2}$

$ix := 1..6$ $iy := 1..6$ $Vxplot_{ix,iy} := v(ix,iy)_0 \cdot 1$ $Vyplot_{ix,iy} := v(ix,iy)_1 \cdot 1$

Dashed lines are ψ and solid lines are ϕ



$\phi, \psi, (Vxplot, Vyplot)$



$\phi, \psi, (Vxplot, Vyplot)$