

**Assignment #4:** MathCAD exercise.

For reference, in MathCAD, imaginary numbers are entered as “1i”, rather than just “I”, and certainly not “1\*i”. A large portion of your grade for this exercise will be on the neatness and organization of your MathCAD sheet, as well as neatness of your final plot.

Airplane wing designers always use complex numbers. The following problem is a starter problem for that kind of work. If you know both the potential function for a flow ( $\phi$ ) and the streamfunction ( $\psi$ ), then you can create a “complex potential”  $w(z)$ :  $w(z) = \phi(z) + i\psi(z)$ . Here,  $z = x + iy$ , as usual. Obviously, you can use this in reverse: if you know  $w$  you can easily compute either  $\phi$  or  $\psi$ . Also, the velocity field is easily found using:  $\frac{dw}{dz} = V_x - iV_y$ . In other words, if you know  $w$ , you know *everything* about a flow in a very compact format.

$$\text{Given: } w(z) = z + 4 \ln(z + 1) - 4 \ln(z - 1)$$

- Enter  $w(z)$ , and have MathCAD determine  $v(z) = \frac{dw}{dz}$ .
- Compute  $v_x(x, y) := \text{Re}(v(x+iy))$ , and  $v_y(x, y) := -\text{Im}(v(x+iy))$ . Then, show the results using the “complex” keyword with the evaluate arrow, e.g.,  $v_x(x,y)$  complex  $\rightarrow$ .
- Compute  $\phi(x, y) := \text{Re}(w(x+iy))$ , and  $\psi(x, y) := \text{Im}(w(x+iy))$ .
- Again, use the “complex” keyword to display  $\phi$  and  $\psi$ . Note that the “arg” function you see here is the same thing as arctangent for complex numbers:

$$\arg(A + iB) = \text{atan2}(A, B) = \text{atan}(B/A)$$

- Make a non-complex version of both  $\phi$  and  $\psi$  using  $\text{atan2}$  instead of  $\text{arg}$ . By “non-complex”, I mean you lose points if there is an “i” anywhere in your result. Obviously, this also means that your final expression may not include any  $\text{Re}()$  or  $\text{Im}()$  functions. Also, there is no reason for your final results to include a “magnitude”  $| |$  function.
- Make a contour plot of  $\psi$  for  $-6 \leq (x, y) \leq +6$ . Use an odd number of grids (such as 99) in quickplot data to avoid the divide by zero problem. Also, use an odd number of contour lines (such as 31) on your plot. It goes without saying that  $\psi$  plots are never, ever colored. When you add the  $\phi$  plot to it, then  $\phi$  can be colored (and should be).
- In one sentence, describe a physical situation that this function  $w$  might represent. If you think this looks like a wing, you need to make an immediate appointment with an eye doctor.