zzPogo, E

p228s10 **Section 4**
Due Wed, May 5, 2010 at 13:00 Set 14
Physics 228: Math Methods CAPA ID 3099

1. [3pt] You toss a coin 7 times. What is the probability that the resulting sequence will be H T T T H H H ?

2. [3pt] You again toss a coin 7 times. What is the probability that the last three tosses will be in the sequence H H H ?

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3. [3pt] A certain die is weighted such that probabilities of showing a 1, 2, 3, 4, 5, and 6 are (3/37), (7/37), (5/37), (3/37), (7/37), and (12/37) . What is the probability of throwing two 2's in succession?

4. [3pt] If this die is thrown twice, what is the probabilty that the first throw will be a 5, and that the second throw will not be a 5?

5. [4pt] If two such dice are thrown, and you are told that the sum of the two is 10 or larger. What is the probability that the result was a pair of 5's?

6. [5pt] How many times would you have to throw this die to have the probability of throwing a 2 exceed 70 percent?

7. [4pt] This die is thrown twice in succession. What is the probability that the first result is odd and that the second result is greater than 4?

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8. [4pt] A group of 14 people all agree that they want to hang out together once per week. Unfortunately, each of them is already scheduled for one other meeting each week. For example, Amy plays softball once per week, Bob plays poker once each week, Cindy volunteers at a shelter once per week, etc. Nobody is willing to meet on the same day as their other meeting. What is the probability that everybody will be available to meet on Saturdays?

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9. [5pt] A weighted coin has a probability 0.72 of showing heads when flipped. This coin is tossed three times. What is the probability that the number of heads minus the number of tails (H-T) is a) equal to -3, b) equal to -1, and c) equal to 0?

10. [5pt] For the same coin as above, what is the probability that the number of heads minus the number of tails (H-T) is a) equal to +1, b) equal to +2, and c) equal to +3?

11. [5pt] This same coin is repeatedly tossed three times, and the number of heads minus the number of tails (H-T) is computed for each set of three tosses. What are a) the mean value of (H-T), b) the variance of (H-T), and c) the standard deviation of (H-T)?

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12. [6pt] Part of the kinetic theory of gases states that the probability of a gas molecule traveling a distance between x and x+dx before colliding with another molecule is

$$ke^{\frac{-x}{\lambda}}\,dx,$$

where λ is a constant which depends on both the type of gas and on the temperature, and k is a constant of proportionality. If $\lambda = 51$ microns, determine a) the value of the constant k, and b) the "mean free path" of molecules in this gas (i.e., the average distance traveled by a molecule before it collides with another).

13. [5pt] Determine a) the probability that an individual molecule of this gas will travel further than 1.9λ before a collision, and b) the probability that it will travel a distance between λ and 1.9λ before a collision.

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14. [5pt] You take a handful of 15 normal pennies, and throw them onto a table. What are the probabilities of a) having 3 of them show heads, b) having 8 of them show heads, and c) having 13 of them show heads?

15. [5pt] For 15 pennies, what are a) the probability of getting fewer than 10 heads, b) the probability of getting at least 13 heads, and c) the probability of getting between 10 and 13 heads (inclusive)?

16. [2pt] For 15 pennies, what is the mean (most likely) number of coins that will show heads?

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17. [5pt] A normal distriubtion has unknown values of μ and σ. What is the probability that the next measurement will be a) within 1σ of μ, b) within 2σ of μ, c) within 3σ of μ, and d) within 1.35σ of μ. Use at least 5 sig-figs.

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18. [6pt] On your way to the bank, you drop a bag of 4100 pennies on the floor. What is the probability that the number of them that are showing heads is between 2040 and 2060 (inclusive)? Use 5 sig figs. Sorry! It doesn't look like MathCAD will be much help on this problem...

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19. [5pt] A certain person recieves an average of 9 email messages every day. In 2010 (which is not a leap year), a) on how many days should this person expect to receive exactly 9 messages, b) on how many days should this person expect to receive less than 9 messages, c) on how many days should this person expect to receive only 1 message? Answers should not be rounded to the nearest integer; you are reporting averages.

20. [5pt] For the same person, a) on how many days should this person expect to receive exactly 14 messages, b) on how many days should this person expect to receive more than 14 messages, c) on how many days should this person expect to receive no messages?

Continued on next page...

21. [5pt] A scientist makes 8 measurements of x and 10 measurements of y:

x data: 7.9 7.8 8.1 8.2 7.8 8.0 8.0 7.7

y data: 1.03 1.00 1.03 0.95 1.01 0.96 1.02 1.05 1.00 0.96

Determine a) the mean value of x, and b) the probable error of x (that is, the 50 percent confidence uncertainty; see page 774).

22. [5pt] Determine a) the mean value of y, and b) the probable error of y.

23. [5pt] Determine a) the mean value of (x + y), and b) the probable error of (x + y).

24. [5pt] Determine a) the mean value of (x*y), and b) the probable error of (x*y).

25. [5pt] Determine a) the mean value of (x^3sin(y)), and b) the probable error of (x^3sin(y)).

26. [5pt] Determine a) the mean value of (ln(x)), and b) the probable error of (ln(x)).

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27. [6pt] A scientist makes 30 measurements of the voltage of a certain solar cell:

3.29, 3.78, 3.85, 3.68, 4.29

2.64, 3.38, 3.73, 3.53, 3.33

3.42, 3.75, 3.40, 3.56, 3.69

3.69, 3.43, 3.07, 3.09, 3.77

3.03, 3.23, 3.65, 4.14, 3.13

3.32, 3.67, 4.26, 3.33, 3.75

For this data, determine a) μ, b) σ, and c) σ_m.

Assign 14

① For 7 tosses, ANY specific sequence has a probability of $\frac{1}{2^7} = \underline{\underline{7.8125 \times 10^{-3}}}$

② for 3 tosses, the probability is $\frac{1}{2^3} = \frac{1}{8} = \underline{\underline{0.125}}$

③ Probs for a die are
$(3/37) \to 1$
$(7/37) \to 2$
$(5/37) \to 3$
$(3/37) \to 4$
$(7/37) \to 5$
$(12/37) \to 6$

two 2's $\to$ $p = \left(\frac{7}{37}\right)\left(\frac{7}{37}\right) = \underline{\underline{0.03579}}$

④ Prob 5 AND $\bar{5}$ is $\left(\frac{7}{37}\right)\left(\frac{30}{37}\right) = \underline{\underline{0.1534}}$

⑤ Sum is over 10 ... possibilities are [illegible]

$$\left.\begin{aligned}
(4,6) &= \frac{(3)(12)}{37^2} = 36/37^2 \\
(5,5) &= 49/37^2 \\
(5,6) &= 84/37^2 \\
(6,4) &= 36/37^2 \\
(6,5) &= 84/37^2 \\
(6,6) &= 144/37^2
\end{aligned}\right\} \text{sum is } \frac{433}{37^2}$$

that sum is 10 or more

Prob (5,5) if we know result is over 10:

$$\frac{\left(\frac{49}{37^2}\right)}{\left(\frac{433}{37^2}\right)} = \frac{49}{433} = \underline{\underline{0.11316}}$$

(6) P (Not a 2) is $\left(\frac{30}{37}\right)$ after n rolls,

$$P(2) = 1 - \left(\frac{30}{37}\right)^n$$

Given $p = 70\% = 0.7$:

$n = 4, \quad p = 56.8\%$

when $n = 5, \quad p = 65\%$

$n = 6, \quad p = 71.6\%$ need 6 rolls

Alternately, $n = \dfrac{\ln(1-0.7)}{\ln(30/37)} = 5.7 \rightarrow$ round up to 6

(7) $p(\text{odd}) = \left(\frac{3+5+7}{37}\right) = \left(\frac{15}{37}\right)$

$p(5 \text{ or } 6) = \left(\frac{7+12}{37}\right) = \left(\frac{19}{37}\right)$

$p(\text{both}) = \dfrac{15 \cdot 19}{37^2} = \underline{0.2082}$

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(8) Each person has a $\frac{6}{7}$ chance of being free on saturday

So, $p(\text{all 14 free on saturday}) = \left(\frac{6}{7}\right)^{14} = \underline{0.1155}$

9 $p = 0.72$ that a coin will be heads.
tossed 3 times
$X = H - T$

Sample Space	Probability	$X = H-T$	P (with 0.72)
HHH	$p \cdot p \cdot p$	3	0.373248
HHT	$p \cdot p \cdot (1-p)$	1	0.145152
HTH	$p \cdot p \cdot (1-p)$	1	0.145152
HTT	$p(1-p)^2$	-1	0.056448
THH	$p^2(1-p)$	1	0.145152
THT	$p \cdot (1-p)^2$	-1	0.056448
TTH	$p(1-p)^2$	-1	0.056448
TTT	$(1-p)^3$	-3	0.021952

$P(x=-3) = 0.02195$
$p(x=-1) = (3)(0.056448) = 0.169344$
$p(x=0) = 0$ (can't be even)
10 $p(x=+1) = (3)(0.145152) = 0.435456$
$p(x=2) = 0$
$p(x=3) = 0.373248$

11 $\mu = \sum x_i \cdot p(x_i) = (3)(0.373) + (3)(1)(0.145) + (3)(-1)(0.056448)$
$+ (3)(0.021952)$

$= 6p - 3 = \boxed{\mu = 1.32}$

$VAR = \sum \left(p(x_i)(x_i - \mu)^2 \right) = -12p(p-1) = \boxed{2.4192 = VAR}$

$\sigma = \sqrt{VAR} = \boxed{\sigma = 1.555}$

(12) As $(b-a) \to 0$, we are told $\int_a^b f(x)\,dx = k e^{\frac{-x}{\lambda}} \cdot (b-a)$

Taking the derivative, we find $f(x) = k e^{-x/\lambda}$

Also, since $\int_0^\infty f(x)\,dx = 1$, we get $-\lambda k \left(e^{-x/\lambda}\right)\Big|_0^\infty = 1$

$= -\lambda k (0-1) = 1$, or $\lambda k = 1$

$\therefore \boxed{k = \frac{1}{\lambda}}$

$\mu = \int x \cdot f(x,\lambda)\,dx$

$\mu = \int_0^\infty x \cdot \frac{1}{\lambda} e^{-x/\lambda}\,dx = -e^{\frac{-x}{\lambda}}(\lambda + x)\Big|_0^\infty = (0 + \lambda)$

$\boxed{\mu = \lambda}$

(13) $\int_{1.9\lambda}^{\infty} f(x)\,dx = -e^{\frac{-x}{\lambda}}\Big|_{1.9\lambda}^{\infty} = e^{-1.9} = \underline{\underline{0.150}}$

$\int_{1.0\lambda}^{1.9\lambda} f(x)\,dx = -e^{\frac{-x}{\lambda}}\Big|_{1\lambda}^{1.9\lambda} = -e^{-1.9} + e^{-1.0} = \underline{\underline{0.218}}$

(14) Tossing 15 pennies. How many are heads?

This is a binomial distribution...

$n = 15$

$p = 0.5$

$q = 1 - p = 0.5$

$$f(x) = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$f(3) = 0.01389$

$f(8) = 0.6964$

$f(13) = 0.00320$

(15) Prob of 0 through 9 is $\sum_{i=0}^{9} f(x) = 0.849$

Prob of 13, 14, or 15 is $\sum_{i=13}^{15} f(x) = 3.69 \times 10^{-3}$

Prob of 10, 11, 12, or 13 is $\sum_{i=10}^{13} f(x) = 0.15039$

(16) $\mu = \sum x_i f(x_i) = 7.50 = \mu$

(17) Normal Distribution. $F = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}}\,dt$

If limits are $(\mu - \sigma)$ to $(\mu + \sigma)$, $p = \mathrm{erf}\left(\frac{\sqrt{2}}{2}\right) = 0.68268949$

If limits are $\mu \pm 2\sigma$, $p = \mathrm{erf}(\sqrt{2}) = 0.95449973 6$

If limits are $\mu \pm 3\sigma$, $p = \mathrm{erf}\left(\frac{3\sqrt{2}}{2}\right) = 0.99730020$

If limits are $\mu \pm 1.35\sigma$, $p = \mathrm{erf}\left(\frac{1.35\sqrt{2}}{2}\right) = 0.82298402$

(18) Binomial with large n → $n = 4100$

$p = 0.5 \quad q = 1 - p = 0.5$

$\mu = n \cdot p = 2050$

$\sigma = \sqrt{npq} = 32.0156$

limits are $X_{min} = 2040$ to $X_{max} = 2060$

$$p = \frac{1}{\sigma\sqrt{2\pi}} \int_{X_{MIN}-0.5}^{X_{MAX}+0.5} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \, dt = \underline{0.25706}$$

(19) Poisson Distribution: $\mu = 9 \qquad N = 365$

$$f(x) = \frac{N \cdot \mu^x}{x!} e^{-\mu}$$

$f(9) = 48.09$

$f(0 \text{ through } 8) = 166.3$

$f(1) = 0.405$

(20) $f(14) = 11.82$

$f(15 \text{ or more}) = 15.135$

$f(0) = 0.04504$

Problem 21 through 26

$$x := \begin{pmatrix} 7.9 \\ 7.8 \\ 8.1 \\ 8.2 \\ 7.8 \\ 8.0 \\ 8.0 \\ 7.7 \end{pmatrix} \qquad y := \begin{pmatrix} 1.03 \\ 1.00 \\ 1.03 \\ 0.95 \\ 1.01 \\ 0.96 \\ 1.02 \\ 1.05 \\ 1.00 \\ 0.96 \end{pmatrix}$$

$xi := 0..7 \qquad yi := 0..9$

$nx := \text{length}(x) \qquad ny := \text{length}(y)$

Averages :

$$\mu x := \frac{1}{nx}\cdot\left(\sum_{xi} x_{xi}\right) \qquad \boxed{\mu x = 7.938}$$

$$\mu y := \frac{1}{ny}\cdot\left(\sum_{yi} y_{yi}\right) \qquad \boxed{\mu y = 1.0010}$$

Probable Errors (p. 774):

$$rx := 0.6745\cdot\sqrt{\frac{\sum_{xi}\left(x_{xi}-\mu x\right)^2}{nx\cdot(nx-1)}} \qquad \boxed{rx = 0.0401829}$$

$$ry := 0.6745\cdot\sqrt{\frac{\sum_{yi}\left(y_{yi}-\mu y\right)^2}{ny\cdot(ny-1)}} \qquad \boxed{ry = 0.0072820}$$

The following results use the same technique of propagation of uncertainty that we always use in Lab, namely that $\Delta f(x,y)^2 = [(df/dx)\Delta x]^2 + [(df/dy)\Delta y]^2 + \ldots$

$$\mu xplusy := \mu x + \mu y \qquad \boxed{\mu xplusy = 8.938} \qquad rxplusy := \sqrt{rx^2 + ry^2} \qquad \boxed{rxplusy = 0.04084}$$

$$\mu xtimesy := \mu x\cdot\mu y \qquad \boxed{\mu xtimesy = 7.945} \qquad rxtimesy := \sqrt{(\mu x\cdot ry)^2 + (\mu y\cdot rx)^2} \qquad \boxed{rxtimesy = 0.07042}$$

$$\mu x3siny := \mu x^3\cdot\sin(\mu y) \qquad \boxed{\mu x3siny = 421.084} \qquad rx3siny := \sqrt{\left(3\mu x^2\cdot\sin(\mu y)\cdot rx\right)^2 + \left(\mu x^3\cdot\cos(\mu y)\cdot ry\right)^2}$$

$$\boxed{rx3siny = 6.69}$$

$$\mu lnx := \ln(\mu x) \qquad \boxed{\mu lnx = 2.072} \qquad rlnx := \frac{1}{\mu x}\cdot rx \qquad \boxed{rlnx = 0.00506}$$

P14A

$$x0 := \begin{pmatrix} 3.29 & 3.78 & 3.85 & 3.68 & 4.29 \\ 2.64 & 3.38 & 3.73 & 3.53 & 3.33 \\ 3.42 & 3.75 & 3.4 & 3.56 & 3.69 \\ 3.69 & 3.43 & 3.07 & 3.09 & 3.77 \\ 3.03 & 3.23 & 3.65 & 4.14 & 3.13 \\ 3.32 & 3.67 & 4.26 & 3.33 & 3.75 \end{pmatrix}$$

The following stuff just converts the 2D array into a 1D array.

$i := 0..29$ $\quad x_i := x0_{\mathrm{mod}(i,6),\,\mathrm{trunc}\left(\frac{i}{6}\right)}$

$$\mu := \frac{1}{30} \cdot \sum_i x_i \qquad \boxed{\mu = 3.52933}\ \boxed{\text{Volts}}$$

$$\sigma := \sqrt{\frac{1}{29} \cdot \sum_i \left(x_i - \mu\right)^2} \qquad \boxed{\sigma = 0.36629}\ \boxed{\text{Volts}}$$

$$\sigma m := \frac{1}{\sqrt{30}} \cdot \sigma \qquad \boxed{\sigma m = 0.06688}\ \boxed{\text{Volts}}$$

x =

	0
0	3.29
1	2.64
2	3.42
3	3.69
4	3.03
5	3.32
6	3.78
7	3.38
8	3.75
9	3.43
10	3.23
11	3.67
12	3.85
13	3.73
14	3.4
15	...