Assignment #4: Mathematica exercise.

This is an exercise in following directions. Recall that in Mathematica, imaginary numbers are entered as "ESC ii ESC", rather than just "i". A large portion of your grade for this exercise will be on the neatness and organization of your Mathematica notebook, as well as the neatness of your final plot.

Airplane wing designers often use complex numbers for 2D flow modeling. The following problem is a starter/teaching problem for that kind of work. If you know both the potential function for a flow(ϕ) and the streamfunction (ψ), then you can create a "complex potential" $w(z)$: $w(z) = \phi(z) + i\psi(z)$. Here, $z = x + iy$, as usual. Obviously, you can use this in reverse: if you know w you can easily compute either ϕ or ψ. Also, the velocity field is easily found using: $\frac{dw}{dz} = V_x - iV_y$. In other words, if you know w, you know *everything* about a flow in a very compact format.

Given: $w(z) = z + 4\ln(z+1) - 4\ln(z-1)$

a. Enter $w(z)$, and have also determine $v(z) = \frac{dw}{dz}$. Note that in Mathematica, ln is "Log".

b. Compute vx(x, y) = Re(v(x+i*y)), and vy(x, y) = –Im(v(x+i*y)). You will need to embed each definition in the function "ComplexExpand[ ]" to get nice results. Show these results.

c. Compute ϕ(x, y) = Re(w(x+i*y)), and ψ(x, y) = Im(w(x+i*y)). Again, use "ComplexExpand[ ]" and show the results.

d. Your ψ answer will include the function "Arg". Manually redefine ψ by hand, replacing Arg with ArcTan in all instances, recalling that:

Arg(A + iB) ≡ ArcTan(A, B)

Your final expressions for ϕ and ψ should not include any imaginary numbers, nor any Im[ ], Re[ ], or Arg[ ] functions.

d. We will define 4 plots:

I. Define a ContourPlot called plot0 that is "black" everywhere, and defined for $-6 \leq (x, y) \leq +6$. Use zero for the function, and use Colorfunction → "AvocadoColors" to color it.

II. Define a ContourPlot called plot1 for ϕ in the same range. Use 25 contours, and "rainbow".

III. Define a ContourPlot called plot2 for ψ in the same range. Use 25 contours. It should go without saying that it is ALWAYS wrong to color a ψ plot in any way, so use ContourShading → None. Also, use Exclusions → None for this plot.

IV. Make a VectorPlot called plot3 in the same range that uses VectorPoints → {13, 13} and VectorColorFunction → None}. Use VectorStyle → {Black, Thick}.

e. Use the Show command to display **only** plot2. You'll notice there is one large, closed, elliptical contour that passes near (0,3) and near (3,0). Use FindRoot[vx[x, 0] = = 0, {x, 3}] and FindRoot[ψ[0, y] = = 0, {y, 3}] to determine the actual coordinates x0 and y0 for this ellipse. You'll have to go back and use this step to create the variables x0 and y0 ABOVE the commands that make the plots. Then, go back to your plot1 definition and add this modification: RegionFunction → Function[{x, y}, 1 < $x^2/x0^2 + y^2/y0^2$]. To see what this did, use the Show command to show **only** plot1.

f. Delete any earlier plot images, and then make a single layered plot of all 4 plots in order, using the Show command.

g. In one sentence, describe a physical situation that this function w might represent. If you think this looks like a wing, you need to make an immediate appointment with an eye doctor.