

Optics and Modern Physics Laboratory

PHYS 226

Fall 2011

Department of Physics and Astronomy
SUNY Geneseo

TABLE OF CONTENTS

| | |
|---|-----------|
| Chapter 0: Introduction | 2 |
| Course Description | 2 |
| Course Materials | 2 |
| Course Requirements | 2 |
| Rules regarding laboratory notebooks: | 3 |
| Error Analysis: | 3 |
| Chapter 1: Index of Refraction of Glass | 7 |
| Chapter 2: Permittivity of Free Space | 11 |
| Chapter 3: Polarization of Light | 17 |
| Chapter 4: Ultrasonic Interference and Diffraction | 21 |
| Chapter 5: The Speed of Light | 29 |
| Chapter 6: The Bohr Atom | 35 |
| Chapter 7: The Michelson Interferometer | 41 |
| Chapter 8: Blackbody Radiation | 47 |
| Chapter 9: Photoelectric Effect | 51 |
| Chapter 10: Chaos – Computer Simulation | 55 |

Chapter 0: Introduction

Course Description

This course introduces students to experimental physics at the intermediate level. Most of the experiments are based on optics, wave phenomena, and modern physics. Some of the topics will be covered in your Analytical Physics III course, but you will also need to use material covered in your Analytical Physics I and II courses.

In this course, emphasis will be placed on good laboratory practice in:

1. performing experiments successfully,
2. recording data,
3. analyzing data, and
4. presenting your work in a neat and coherent manner.

Course Materials

- Your calculus-based Analytical Physics I textbook (e.g., *Fundamentals of Physics*, by Halliday, Resnick, and Walker).
- Your Analytical Physics III textbook (e.g., the modern physics textbook by Serway or by Thornton and Rex).
- A **hard** covered, quad-ruled laboratory log notebook.
- A 3-ring binder for this lab manual. You will download sections of this manual each week, which you must have in your binder at the beginning of each lab.

Lab Books

Your logbook should contain the following information:

- **Pre-lab Work:** You will be required to research each topic before coming to lab each week. The pre-lab assignment will give some basic guidelines for study, and will include a few specific questions. Record your own preparatory notes (explanations, sketches, equations, etc.) in your notebook before you come to lab, including (but not limited to) the answers to the questions.
After your own preparatory material, you may also include notes from any pre-lab discussion provided by your instructor.
- **Equipment:** List the equipment and supplies you needed to perform the experiment. This is a shopping list if you were to ever repeat the experiment. Include specific model information when available, in case you have problems with or questions about the equipment.
- **Diagrams:** Neatly sketch and label the apparatus as you use it. You may include more than one sketch, as needed.
- **Procedural Record:** You should make a record of events *as they actually happen* in the lab. Besides describing what finally worked, include any false starts, equipment problems,

mistakes etc. Discuss work that was more difficult than expected, and why. Attach computer outputs, graphs, photos, etc. in your lab notebooks in chronological order. Annotate them as required. Do not attach pages that the reader must unfold to view. Tables, when used, should be neat, titled, and labeled. Use SI units (you will be tempted to record data in inches occasionally). Be especially sure to record any insights or ideas you have as soon as you think of them.

- **Calculations and Analysis:** All calculations must be done in your notebook. Note that calculations are not the same thing as numbers. The purpose of including calculations is to document the intermediate steps, rather than just the final result. Include units throughout. “Analysis” includes comments on whether a result is reasonable. Also, quantitative analysis of sources of error should be done here. In other words, for each potential source of error, compute the potential numerical impact on your result.
- **A Summary Table:** Create a table of all your final numerical results for the experiment.
- **Abstract:** Write an abstract for the entire experiment. Abstracts must fit on one page. Include your purpose, your procedure, a discussion of the analysis you performed, general comments about the experiment, possible sources of errors, your numerical results, and your conclusions. Your comments should include a discussion of how your chosen procedure and analysis method compare to alternate methods. Abstracts must be typed and then stapled into your notebook.

Other Rules for Laboratory Notebooks:

1. Page numbers for the *entire* notebook must be written on every page *on the first day of lab*. Number (and use) both sides of each sheet. **Leave pages 1 and 2 blank for use as a table of contents, to be filled in as the semester progresses.**
2. *Never* remove any sheets from your notebook.
3. You may never erase any entry in your notebook. Mistakes are to be crossed out with a single horizontal line, so that they remain legible. Write a *short* explanation next to each error, and then proceed with the correct work immediately below the error.
4. Use the laboratory notebook to write down all relevant information. You may never use loose sheets of paper. Your grade will be penalized any time you copy work into a notebook at a later time. Computer generated printouts, when needed, must be neatly attached to the pages of your notebook on all four edges using staples or tape. You may not attach documents that are larger than a single page of your notebook. Any loose material turned in with your notebook will be **discarded**, ungraded.
5. Do not copy or paraphrase sentences from this manual. Use your own words to describe what you did in the lab.

Error Analysis:

The following is adapted from “Analytical Physics 124 Laboratory Manual,” by K.F. Kinsey and J.D. Reber:

There are a few quantities in science that are exact either because they are mathematically defined (such as π and e), or which are physically defined (by agreement) as the basis of measurement, such as the second, the speed of light, etc. Everything else has to be measured and in the process of

measuring, there is always some limit to the accuracy of the measurement, no matter how careful the experimenter or how expensive the equipment. No measurement can yield the “true” or “actual” value of the quantity being measured; even the best result is only an estimate of that value within the limitations imposed by the measuring process. This estimated value can be interpreted properly only if we know the limitations on the measurement, i.e., *the uncertainty*.

Note this very important point: there are no books in which we can look up the “true” value of a physical quantity. Any value listed in a reference book is the result of somebody’s measurement and will possess some uncertainty, even if this uncertainty is not provided. Presumably, the value quoted will be the result of the most accurate experiment to date, and the uncertainty will be relatively small. In the philosophical sense, there presumably exists a “true” value, but we will *never* know what it is. There will *always* be an uncertainty: given time and patience, we may be able to make the uncertainty smaller, but we can never reduce it to zero.

It is also important to realize that when we are dealing with uncertainties, we are forced to realize that we are talking about *probability*. By the very nature of uncertainties, we can never know *precisely* how much our measured value may depart from the “true” value. By convention, in the “normal” case, we report uncertainties such that the “true” value has about a 68% probability of being within our reported range.

In science, the term “error” is sometimes used equivalently to “experimental uncertainty”. “Error” should not be thought of as some sort of mistake which can be corrected.

Some of you, in former courses may have been taught that experimental error is described as the percentage difference between your result and the “accepted value”. But in this course, we will not assume any “accepted value”. Consider your experimental errors as if your experiment were the first and only time that quantity has ever been measured. So, whatever you measure *is* the accepted value as far as this lab is concerned. Since nobody will ever pay you to measure a quantity for which there is already an “accepted value”, this attitude will help prepare you to perform real science outside of the classroom.

Propagation of Error

We will use the notation “ Δx ” to indicate the uncertainty in the value x . Usually, an experimentally measured quantity $x \pm \Delta x$ is not the *end result* of our experiment, but merely one quantity which contributes to a final result, say F . At some point, the value for x will be plugged into an expression for the final result F . Therefore, any uncertainty Δx in our measured quantity x will also contribute to the uncertainty ΔF .

In general, F will depend on many variables, e.g., $F = F(x, y, z, \dots)$. Each of the variables (x, y, z, \dots) has both a value and an uncertainty. In the simple case where F depends on only one variable x , the uncertainty ΔF is obtained from the following:

$$\Delta F = \left| \frac{dF}{dx} \Delta x \right|.$$

As an example, the quantity F might be the area of a circle (which might more sensibly be labeled “ A ”), and the measured quantity x might be the radius (which might more sensibly be labeled “ r ”). Since the area A is given by $A = \pi r^2$ the uncertainty in A , using the above equation, is:

$$\Delta A = |2\pi r \Delta r|.$$

The extension of this expression to the case where F depends on many variables is straightforward. The same is true for each of the variables as was true for x : each variable has a *value* and an *uncertainty* which contributes, in its own way, to the final value F and its uncertainty ΔF , respectively. The resulting uncertainty ΔF is a combination of all the individual variable uncertainties:

$$\Delta F = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial F}{\partial y}\right)^2 (\Delta y)^2 + \left(\frac{\partial F}{\partial z}\right)^2 (\Delta z)^2 + \dots}$$

A partial derivative (e.g., $\partial F/\partial x$) is nothing more than a derivative of F with respect to x alone, treating all other variables (y, z, \dots) as constants. Note that F and ΔF must have the same units.

As an example, imagine that the final quantity in which we are interested (i.e. F) is the volume of a cylinder (which might more sensibly be labeled “ V ”), to be computed from a measured value of its radius r and its height h . Because the volume is given by $V = \pi r^2 h$, the individual contributions to ΔV (the uncertainty in V) from the two variables r and h , using the above equation, are:

$$\Delta V = \sqrt{(2\pi r h)^2 (\Delta r)^2 + (\pi r^2)^2 (\Delta h)^2}.$$

Once again, in our artificial environment that we call the laboratory, we frequently measure *known* quantities. The main purpose for measuring known quantities is to train you to both conduct experiments and assess measurement errors. By measuring a “known” quantity you can see how your techniques measure up to those of experienced scientists with very sophisticated equipment. If you have performed your experiment carefully, you would expect there to be some overlap between your value and the “accepted value”. That is, *your value* \pm *your error* and the *accepted value* \pm *accepted error* would include some common ground.

Example: $8.72 \pm 0.64 \text{ m/s}^2$ “agrees with” $9.25 \pm 0.12 \text{ m/s}^2$, but $8.72 \pm 0.23 \text{ m/s}^2$ does not agree. Because of the probabilistic nature of errors, actual overlap is not *necessary*, but if the values differ by more than twice the error, it is unlikely that they agree.

Percent Errors: Many students are in the habit of reporting “percent errors”. This practice can be highly misleading. Consider two measurements: $200.0 \pm 1.0 \text{ cm}$ and $1.0 \pm 0.10 \text{ cm}$. The first would

have 0.5% error while the second would have 10% error, yet the second is clearly a more precise measurement. The situation would be even more misleading if these were measurements of *position* of an object, where the value depends on an arbitrarily chosen zero position. Choose a different zero and the “percent errors” will change. **For this course, do not report “percent errors”.**

Error Format: The following error format is standard and is required for your notebooks:

- a. Round off uncertainties to two significant figures. For example, ± 0.2372 would become ± 0.24 .
- b. The best value is to be rounded off to the same decimal place as the error. For example, 12.38243 ± 0.2372 would become 12.38 ± 0.24 .
- c. The value and its uncertainty *must* share the same units and the same exponent. For example, 7856429 ± 8734 s should be written as $(7.8564 \pm 0.0087) \times 10^6$ s or (7.8564 ± 0.0087) Ms. Note that as shown in both examples, it is preferable that the uncertainty be written in true scientific notation such that there is exactly one significant digit before the decimal place.
- d. A value may never start with a decimal point. So, 0.65 is correct, but .65 is incorrect.
- e. Do **not** use computational notation (i.e., never use the symbols E, *, or ^) for your written work; the following are both **incorrect**: $(8.50 \pm 0.20)E3 \Omega$, and $(8.50 \pm 0.20) * 10^3 \Omega$. Use $(8.50 \pm 0.20) \times 10^3 \Omega$ or $(8.50 \pm 0.20) \text{ k}\Omega$ instead.
- f. Use the actual \pm symbol. Don't write out “plus or minus” or “+/-” or “+ or -”, etc.

Chapter 1: Thin Lenses and Index of Refraction

Overview:

In this experiment we will examine the refraction of light through glass lenses (“geometrical optics”). We will determine the index of refraction of the glass, and use this to compute the speed of light in glass.

Suggested Reading Assignment:

The section on refraction of light in the chapter about “Electromagnetic Waves”, and the sections on lenses in the chapter about “Images” in your Physics I textbook.

E.g., Section 33-8, and Sections 34-6 through 34-7 of Halliday, Resnick, and Walker, 7th edition.

Pre-lab Questions:

1. How are focal length f , image distance d_i , and object distance d_o related? When is each negative? Contrast f for concave lenses with convex lenses. Which is converging? How does a “virtual image” differ from a “real image”?
2. What is the Lensmaker’s equation? When is either radius of curvature negative? For a spherometer (see next pages) what is R as a function of h and L ?
3. For a typical ray diagram, there are an infinite number of rays that *could* be drawn, but we typically only draw three of them. Describe these three rays.
4. There are three possible configurations for each type of lens, depending on where the object is placed relative to the focal point (i.e., in front of, at, or behind the focal point). Draw a ray diagram to scale for each of the six cases. Note that “to scale” always means “proportionally correct”, rather than “life size”. Choose numeric values for the focal length, object height, and object distance for each case and determine:
 - a) whether the image is real or virtual,
 - b) the magnification of the image, and
 - c) whether the image is upright or inverted.

Compare the measurements of your scale drawings with the computed predictions of the thin lens equation. How can the magnification m be determined from d_o and d_i ?

5. An object of height $h = 6$ cm is located at $x_{o1} = 5$ cm. A lens ($f_1 = +10$ cm) is located at $x_{L1} = 30$ cm. A second lens ($f_2 = -15$ cm) is located at $x_{L2} = 40$ cm. The image of the first lens is the object for the second. Find: d_{o1} , d_{i1} , x_{i1} , x_{o2} , d_{o2} , d_{i2} , x_{i2} , m_1 , m_2 , and m_{total} . Draw a ray diagram to scale for this problem.
6. What is the smallest possible value for the index of refraction? What is the index of refraction for air? For water? For an ordinary glass? For polycarbonate plastic?
7. What is the value for the speed of light in vacuum? What is the relationship between the speed of light and the index of refraction?

Objectives

Our objective is to determine the focal lengths of converging and diverging lenses using geometrical optics. From these data and from the geometry of the lenses we will then calculate the index of refraction of the material the lenses are made from.

Equipment List

- Converging and Diverging lenses
- Optical bench, lens holders, screens, etc.
- High intensity lamps, flashlights
- Spherometer

Procedure and Analysis

Part A: Converging lens

Method 1: Choose a converging lens and first determine its focal length roughly using a distant object as the source (it will be difficult to use any indoor object). Draw a ray diagram. Since the object is so far away compared to the diameter of the lens, all of the rays that reach the lens are virtually parallel. Parallel rays from the object will converge towards the focal point after they pass through the lens. If you can focus the image on a screen, then the distance between the screen and the lens is the focal length. Because it isn't easy to be sure an image is *perfectly* in focus, repeat this experiment several times, and take the average of your trials. This gives you a preliminary result for the focal length.

Method 2: Next, using the light box, lens and screen, measure experimentally the image distance d_i for ten different values of d_o , the object distance. Also, record the object and image height for each position. **Be careful!** In the thin lens equation, each *distance* is measured from the center of the lens, but in your experiment, each *position* is measured with respect to a coordinate axis! Be sure that measurements on your x -axis increase from left to right (as usual). Draw a ray diagram. From a plot of your data determine the focal length of the lens. For each position, compute the magnification using the ratio of heights as well as the ration of distances, and compare the two with a plot. How well do they agree?

Part B: Diverging Lens

For a diverging lens the image is “virtual”, which means that it cannot be seen on a screen. So, we will need to use a combination of lenses to examine the concave lens. We will use the image from a converging lens (the same one you used earlier) as the virtual object for the diverging lens. When the lenses are positioned appropriately, a real image can be created. Before setting up for this part of the experiment, draw a ray diagram for your known value of f_1 and predict the location of the image.

On the optical bench, first set up an object and image using the *converging* lens. The image of this lens will serve as the object for the diverging lens. Now place the diverging lens between the convex lens and its image. Keeping the light source and the converging lens in place, move the diverging lens (and screen) only for your data. Measure the screen position when the image is focused. Record data for eight different lens positions. Again, from a plot of the data determine the focal length of the concave lens. Again, be careful: image and object distances are not the same as image and object positions!!

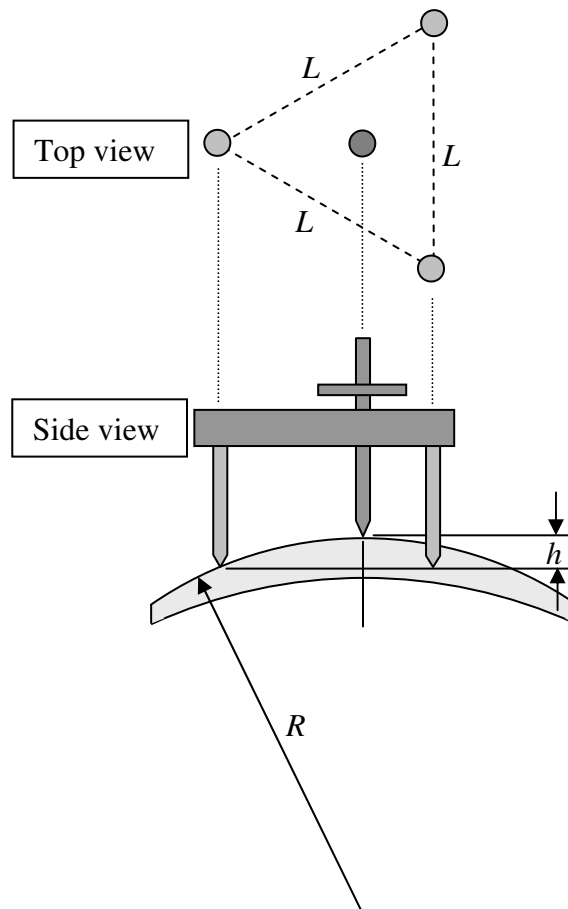
Also for each lens, measure the radii of curvature of each surface using a spherometer. You will probably also need to use the spherometer on a flat surface to determine the true zero point. Then, use the Lens Maker's Equation compute the refractive indices for each lens. Look up the refractive index of glass in the CRC manual and comment on your results.

Again, as you analyze data, keep in mind that your lenses aren't at $x = 0$ on your optical bench, so, for example, $d_o \neq x_{\text{object}}$.

A complete error analysis is required.

Spherometer

The spherometer has three legs in an equilateral triangle of side L , and an adjustable center post. Once it is properly zeroed on a flat surface, the indicated vertical height h , along with the measured length L , can be used to compute the radius of curvature of the surface it is resting on. Note that h is positive for convex surfaces, and negative for concave surfaces.



Theory

A converging lens (f is positive) is usually convex on both sides (thicker in the middle than at the edges) and a beam of incoming rays parallel to the axis converge after passing through the lens. A diverging lens (f is negative) is concave on at least one side; parallel rays diverge after passing through this type of lens.

The relationship between the object distance d_o , the image distance d_i , and the focal length f is given by

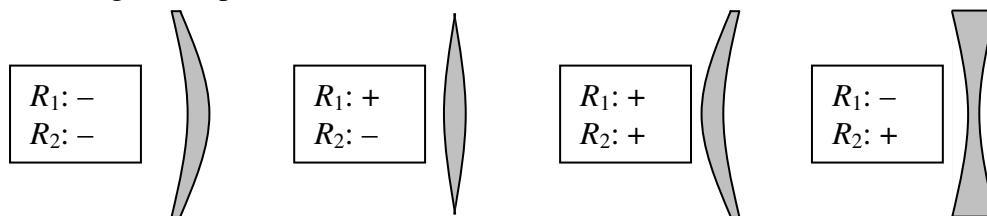
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

The magnification is given by $m = \frac{h'}{h} = -\frac{d_i}{d_o}$,

where h and h' are respectively, the object and image heights. For a thin lens there exists a relation between its focal length and the refractive index in terms of radii of curvature of the surfaces, given by

$$\frac{1}{f} = (n - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right),$$

where r_1 is the radius of curvature of the lens surface on which the light falls first and r_2 is that of the second surface. This equation is known as the *Lens Maker's Equation*. Either r may be positive or negative, depending on whether the center of curvature is on the left (r is negative) or the right (r is positive) of the lens:



For the spherometer, the radius of curvature is found from the Pythagorean Theorem to be:

$$|R| = \left| \frac{L^2}{6h} + \frac{h}{2} \right|.$$

You will need to adjust the sign manually, since the sign of h depends on whether the individual surface is convex or concave, whereas the sign of R depends on the orientation of the lens.

Chapter 2: Permittivity of Free Space

Overview:

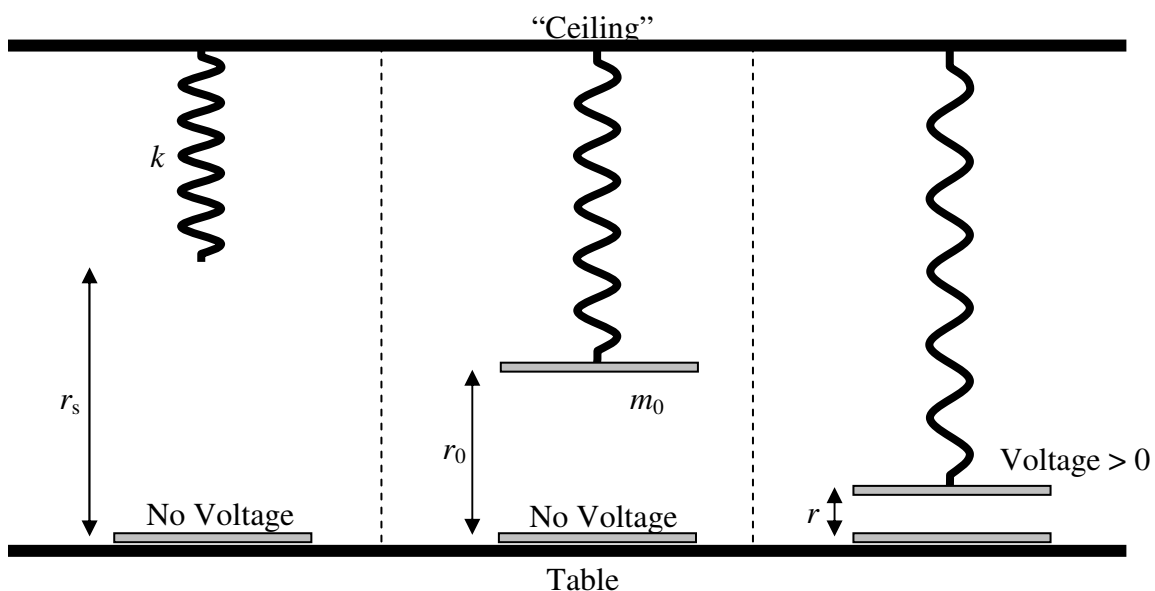
Every charged particle creates a distribution of electric field. At any point in space, the magnitude of the electric field depends not only on the charge and its position, but on the permittivity of the space around the charge and your chosen point. This is true for electric fields in vacuum (“free space”) as well as for those in various materials. We will use the concept of capacitance to attempt to measure the permittivity of air. As a check, we will combine our resulting permittivity with the known magnetic permeability to determine the speed of light.

Suggested Reading Assignment:

Chapters titled “Capacitance” and “Electromagnetic Waves” in your Physics I textbook. e.g., Chapters 26 and 34 of Halliday, Resnick, and Walker, 6th edition.

Pre-lab Questions:

1. What is the relationship between the speed of light, the permittivity, and the magnetic permeability? What are the accepted values of these constants in vacuum? What is the uncertainty of each of these three values?
2. How much do these three constants differ for air as compared to vacuum?
3. What is the force between the capacitor plates of a parallel plate capacitor, assuming that their separation is much smaller than their size? Express this force as a function of ϵ_0 , A , the area of the plates, V , the voltage between the two plates, and r , the distance between the plates.
4. In the figure below, an upper plate, having weight m_0g , is made to hang from a spring having constant k . Use free-body analysis to obtain an equation for V as a function of r . Your final answer may include r_0 , but neither r_s nor m_0 . Plot this function. Where is the maximum of this function located?



Objectives

Our objective in this experiment is to determine an experimental value for the permittivity of free space (despite the fact that it is a defined quantity with no uncertainty, like π).

Equipment List

- Electrostatic balance
- Milligram weights
- High voltage power supply (0 to 400-700V)
- Timers

Procedure and Analysis

The experimental set up is a basic circuit connecting the power supply and voltmeter to the electrostatic balance. Put a small weight (about 0.5 grams) on the upper plate. Place the spring support so that the upper plate is about 0.5cm above the lower plate. Put the lower plate directly underneath the upper plate aligning them as best as you can. Position the weights to make the plates as parallel as possible. This is very tricky. Also, make sure that the plates are aligned so that their areas overlap completely. This can be done by rotating the lower plate.

Turn the power supply on and slowly increase the voltage. Do not shift or adjust or add weights with the voltage on! The plates should come together in the upper 25% of the voltage range of the power supply. For the remainder of the experiment ignore the weights you just added. This weight will be considered to be part of m_0 . The above steps were done to ensure that the plate separation is small enough to bring the plates together when the voltage is in the correct range.

Now begin taking data. First, with no additional weights, start the voltage at zero and estimate the distance R (you'll confirm it later using Eq. [1]). Then, slowly increase the voltage until the plates come together. Record V_{\max} for this case (zero added weight; i.e, for $m' = 0$). Remember that this is the lowest voltage that makes the plates come together. This is a fairly delicate measurement. Next, turn down the power supply, and then turn it off.

Then add some weight (in increments of around 50 mg), and repeat the above measurement, up to about ten data points. From a plot of this data we can determine the permittivity, assuming that the spring constant is known.

To determine the spring constant we will perform a separate experiment starting with about 20g (and with increments of 20g) for m' and measuring the time period of oscillations using the timers. With about eight data points, the spring constant can be determined from a plot of this data (see Eq. [2]).

You will need to measure the area of the lower plate. All quantities should include uncertainties.

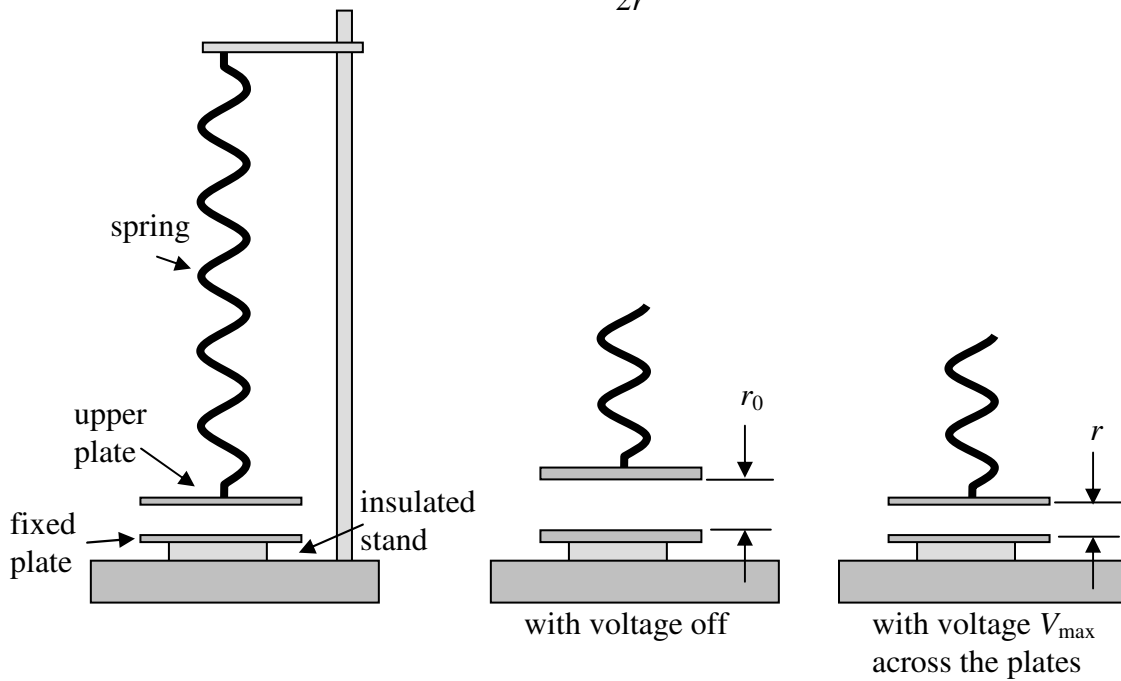
Some things to consider:

1. How accurate is the voltage measurement?
2. If the capacitor plates are not exactly parallel how does it affect the experiment?
3. The speed of light in vacuum is related to permittivity and permeability of free space, i.e., vacuum. What effect does the presence of air have on your results?

Theory

The main component of the apparatus is an electrostatic balance which is comprised of a conducting plate hanging by a spring over another identical plate, the two plates forming a parallel plate capacitor. The electrostatic force on the upper plate is approximately given by

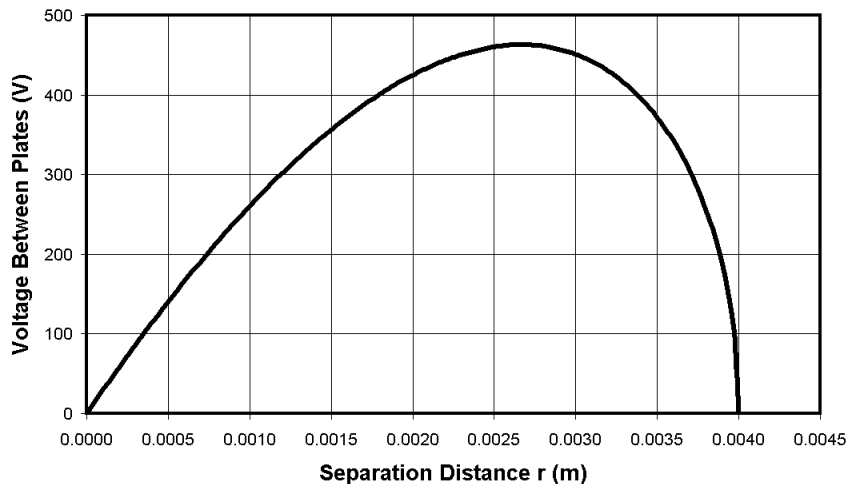
$$F = \frac{\epsilon_0 AV^2}{2r^2}.$$



At equilibrium, the force between the capacitor plates is equal to the restoring force in the spring, i.e.,

$$k(r_0 - r) = \frac{\epsilon_0 AV^2}{2r^2}.$$

Effect of Voltage on Plate Separation
 Assumes $R = 4 \text{ mm}$, $k = 1.2 \text{ N/m}$, and $A = 120 \text{ cm}^2$



The maximum value for the potential, V_{\max} occurs at $r = 2r_0/3$. As V is increased from zero to V_{\max} , r goes from r_0 to $2r_0/3$. If the voltage is increased beyond V_{\max} , there is no value of r for which the restoring force of the spring can balance the electrostatic force. The plates will move as close together as possible, coming together. In the experiment they may come together slightly sooner because of the always-present vibrations of the upper plate.

After some algebra, we obtain the equation relating r_0 and V_{\max} ,

$$r_0 = \frac{3}{2} \left(\frac{\epsilon_0 A}{k} \right)^{1/3} (V_{\max})^{2/3}.$$

In the experiment, the voltage V_{\max} will be measured for many values of r_0 . The distance r_0 is changed by adding small weights to the upper plate. From the force balance we have

$$m'g = k(R - r_0),$$

where $R = r_0$ for no additional mass. Here, m' is only the small mass added to the plate, and does not include the mass of the plate itself. Substituting the expression for r_0 above, we obtain

$$m' = - \left[\frac{3k^{2/3}}{2g} (\epsilon_0 A)^{1/3} \right] (V_{\max})^{2/3} + \frac{kR}{g}. \quad [1]$$

The maximum voltage is measured for different masses. The permittivity of free space can be determined from a plot of this data, if the spring constant k is known.

The spring constant can be determined independently by measuring the time period of vertical oscillation of the upper plate with different masses. The period is given by

$$T = 2\pi \sqrt{\frac{m_0}{k}},$$

where m_0 is the mass of the upper plate plus one-third of the mass of the spring. We will, in fact, eliminate it by doing the following. When extra mass is added to the upper plate the equation for the time period becomes

$$T' = 2\pi \sqrt{\frac{(m_0 + m')}{k}},$$

where m' is the amount of added mass. The above equations can be algebraically manipulated to become

$$T'^2 - T^2 = \left(\frac{4\pi^2}{k} \right) m'. \quad [2]$$

A plot of the square of the measured values of time periods for different values of m' will yield a straight line, whose slope is related to the spring constant.

Chapter 3: Polarization of Light

Overview

In this experiment we will study Malus' Law: the dependence of the intensity of light when passed through a set of polarizers on the relative orientation of the polarizers.

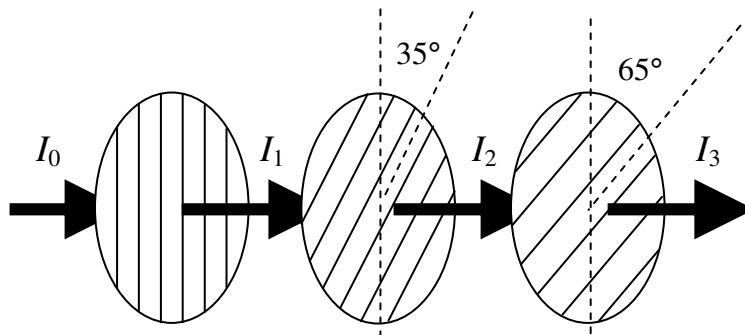
Suggested Reading Assignment

The section on "Polarization" in your introductory physics text.

E.g., Section 33-7 of Halliday, Resnick, and Walker, 6th edition.

Pre-lab Questions

1. What is polarization?
2. Is sunlight polarized? What about sunlight that reaches the surface of the earth?
3. What is "Malus' Law" (Halliday, Resnick and Walker call this the "cosine-squared rule")? Under what circumstances is this law obeyed?
4. Consider unpolarized light of intensity I_0 that passes through 3 polarizers, as shown below. Compute the intensities I_1 , I_2 , and I_3 in terms of I_0 .
5. Explain how a polarizer and an analyzer can be arranged so that no light exits the analyzer.



Objectives

Our goal is to verify Malus' Law. In particular, we want to verify that polarizers affect light intensity following a cosine squared law.

Equipment List

- polarizer and analyzer
- Incandescent Lamps
- light meters

Procedure and Analysis

This simple experimental set-up involves an incandescent lamp followed by a pair of polarizers and the illumination meter. One of the polarizers plays the role of the “polarizer” in the sense that it polarizes the incident light, and the other will be the “analyzer”. Now change the angle between the polarizer and analyzer by rotating the polarizer only in steps of 10° . Start taking data at 5° and then proceed in one direction to 355° .

We will analyze the data using two different methods.

Method I: Use the “Solver” tool in Excel to fit your data using this form:

$$I(\theta) = (I_{\max} - I_{\min}) \cdot |\cos(\theta - \theta_0)|^n + I_{\min}$$

Note that you will need to use Solver to determine 4 parameters: I_{\max} , I_{\min} , θ_0 , and n . Also, determine Δn .

Method 2: With some manipulation, your data can also be used to determine the value of n using a straight line fit. First, we rearrange the above equation as follows:

$$\frac{I(\theta) - I_{\min}}{I_{\max} - I_{\min}} = \cos^n \theta.$$

Then, we take the natural log of both sides of this equation:

$$\ln\left(\frac{I(\theta) - I_{\min}}{I_{\max} - I_{\min}}\right) = \ln(\cos^n \theta) = n \ln[\cos(\theta)]$$

Note: if $\cos(\theta) < 0$, then you should replace it with $\cos(\theta + \pi)$.

What should you plot to determine n ? Is your value of the power close to the expected value? Is it within experimental uncertainty? What are some of the sources of error in this experiment?

Theory

When light passes through a set of polarizers (the first being called the polarizer and the second the analyzer), its intensity is a function of the angle θ between the polarizing directions of the polarizer and the analyzer. This relationship, discovered by Malus, is given by

$$I(\theta) = I_{\max} \cos^2 \theta ,$$

where I_{\max} is the maximum value of the transmitted intensity. According to this law light should be completely cut off at $\theta = 90^\circ$. However, in practice the intensity is not exactly equal to zero at $\theta = 90^\circ$. In order to account for this the above equation can be modified to read:

$$I(\theta) = (I_{\max} - I_{\min}) \cos^2 \theta + I_{\min} ,$$

where, $I_{0^\circ} = I_{\max}$, and $I_{90^\circ} = I_{\min}$. Substituting all of this in the above, we have

$$\frac{I(\theta) - I_{90^\circ}}{I_{0^\circ} - I_{90^\circ}} = \cos^2 \theta .$$

We will be using this version of Malus' law in our analysis.

Chapter 4: Ultrasonic Interference and Diffraction

Overview:

In this experiment we will study interference and diffraction of ultrasonic waves. Interference and diffraction occur when waves from two sources superimpose in a region of space.

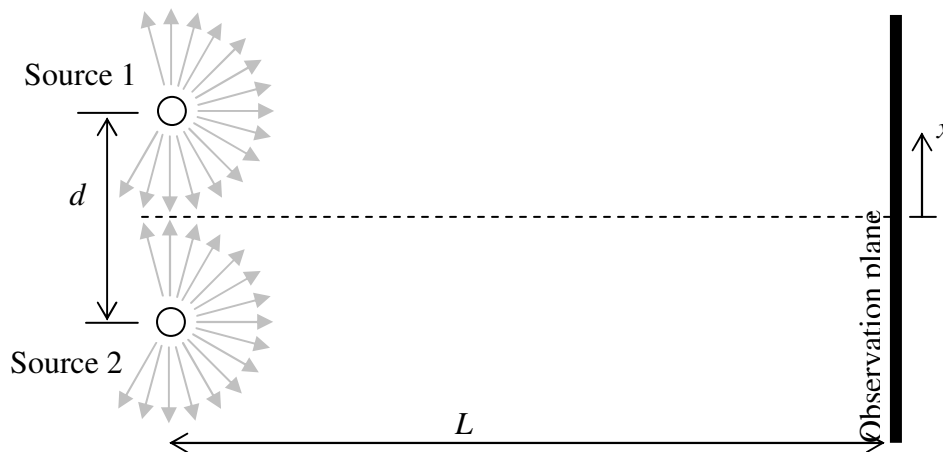
Suggested Reading Assignment:

Chapters on “Interference” and “Diffraction” from an introductory physics book. Although these chapters focus primarily on light waves, the theory is the same for sound waves.

E.g., Chapters 36 and 37 of Halliday, Resnick, and Walker, 6th edition.

Pre-lab Questions:

1. Under what conditions will two sources “interfere”?
2. Define “coherence”.
3. For two wave sources (see sketch below), find an equation for the intensity at any point x in the observation plane. Plot the intensity vs. x , and identify a few maxima and minima. Where is the 3rd maxima? The 12th?
4. For a single wave source of diameter a , find an equation for the intensity at any point x in the observation plane. Plot the intensity vs. x , and identify a few maxima and minima. Where is the 3rd minima? The 5th?
5. For two waves sources of diameter a , separated by a distance d , determine the intensity at any point x in the observation plane. Plot the intensity vs x . Suppose that a is somewhat smaller than d . How do changes in a or d affect the resulting intensity pattern?



Objectives

When two sources of finite cross-sectional area are driven in phase, we expect to find a combination of interference and diffraction patterns. Our objective in this experiment is to map out the pattern formed by ultrasonic waves from two speakers by measuring the intensity received by a detector some distance away. From the interference pattern we will determine the distance between the sources, and from the diffraction pattern we will determine the diameter of the sources.

Equipment List

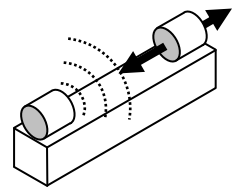
- Ultrasonic transducers
- Function Generator
- Oscilloscope
- Optical bench
- Clamps

Procedure and Analysis

Week 1

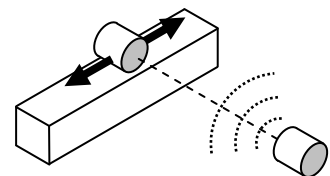
(1) *Measure speed of sound, study how amplitude of signal falls off with distance.*

Place two transducers facing each other on the optical bench, close to one edge. Be sure to put the two transducers at the same height on the bench. Plug the source into channel one and the receiver into channel two on the scope. Set the function generator to produce its maximum amplitude sine wave. Carefully adjust the frequency of the function generator to maximize the amplitude of the receiver signal. Use the scope to record the frequency of the signal. Slide the receiver along the optical bench and count the number of wavelengths on the scope that pass by. Stop every 5 wavelengths or so and record the position of the receiver on the optical bench, and also record the peak to peak voltage of the received signal. To get the most accurate measurement allow the receiver to move completely to the other side of the optical bench. Use your results to find the speed of sound (with uncertainty). Plot the peak to peak voltage versus receiver position. Does the peak to peak voltage fall off like $1/r$ or like $1/r^2$? Justify your claim. Now plot the intensity (in arbitrary units, but proportional to mV^2) of the signal as a function of receiver position. Does the intensity of the signal fall off like $1/r$ or like $1/r^2$? Use Solver to justify your answer (using a fit of the form $I = A(x - x_0)^{-n}$. Solver will give you three parameters, but you only care about n .



(2) *Single source diffraction:*

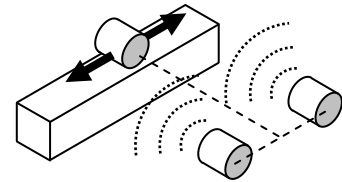
Set the source transducer on one side of the table. Put the source up high enough so that it is roughly halfway between the table and the ceiling. Place the receiving transducer on the optical bench. Use a long enough rod so that the receiver is at the same height as the



source. Placing the transducers up high will minimize unwanted reflections of the sound source off of the table. Set it up so that the center of the optical bench is directly across from the source. Take special care to ensure that the source transducer is facing directly across the table. Make sure it doesn't angle up or down, or left or right. Again tune the source frequency to maximize the amplitude of the receiver signal. Move the receiver in two centimeter increments along the entire length of the optical bench. Record the peak to peak voltage of the receiver signal for each position. Plot the intensity of the signal (in arbitrary units) versus position. Fit your data with an appropriate function. Include both diffraction effects and $1/r^2$ effects. Hold the frequency, the wave speed, and the distance from source to optical bench constant, and allow the aperture size, receiver position offset, maximum intensity and intensity offset to vary to optimize the fit. Perform an uncertainty analysis on the aperture size. How does the aperture size from the fit compare with the actual size of the transducer?

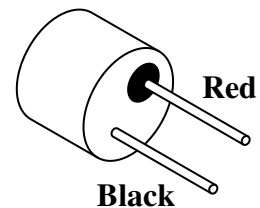
Week 2

(3) *Interference/Diffraction:* Use the same setup as last week, but now use two sources and one receiver. Make sure you drive the two sources in phase (not out of phase). Set it up so that the line joining the two sources is parallel to the optical bench axis. Set up the optical bench so that its center is directly across from the midpoint between the two sources. Separate the two sources approximately by a distance (in meters) equal to 2500 divided by the frequency of the signal in Hz. For example, if you have a 25 kHz transducer, use a separation of about 10 cm. Place the receiver at the center of the optical bench, at a position directly across from the midpoint between the two sources. Note how a small displacement forward or backward of one of the sources affects the amplitude of the receiver signal. Make sure that the optical bench is parallel to the line between the two sources. Take special care to ensure that the two source transducers are both facing directly across the table, rather than upwards, downwards, left, or right. Carefully measure the distance between the centers of the two sources; be careful not to bump the sources. Slide the receiver back and forth and note how the amplitude of the receiver changes. Make sure things seem to be working properly. Now you are ready to take data. Start at one end of the optical bench and move the receiver in 5 mm increments all the way to the other side. Record the amplitude of the receiver for each position.

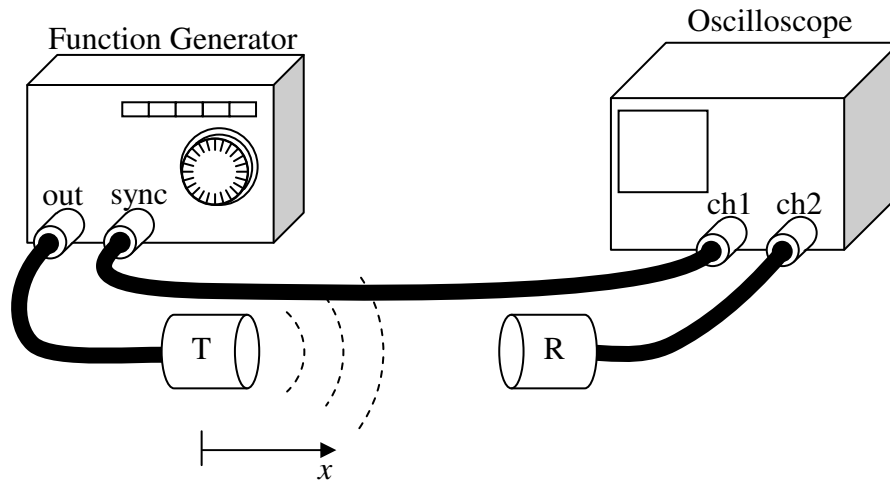


Hardware

Although they look the same, some of the pressure transducers are intended to be Transmitters, and some are intended to be Receivers. You can tell which is which by the last letter (T or R) in the part number on the back. Also, each has two wires. The main signal wire (red) should connect to the insulated input (surrounded by black rubber). The black ground wire should go to the wire that touches the transducer casing.



There are many satisfactory ways to connect the function generator and oscilloscope for part one. Here is one way:



Analysis: Plot the intensity of the sound wave as a function of receiver position. The pattern should be a combination of interference, diffraction, and the $1/r^2$ effect. There is a good chance that there will be a systematic error in your x values, due to your inability to center the detector with respect to the two sources. Also, you will notice that there is a background intensity due to other noise in the room.

Use the speed of sound you determined last week and the frequency from the function generator to find the wavelength. Hold the frequency, wave speed, and distance from the center of the source to the optical bench constant, and allow the distance between the sources, the size of the sources, the optical bench position offset, the maximum intensity, and the intensity offset to vary to optimize the fit. Perform an error analysis on the distance between the sources (d) and the size of the sources (a). How do the distance between the sources and the size of the sources compare to the directly measured values?

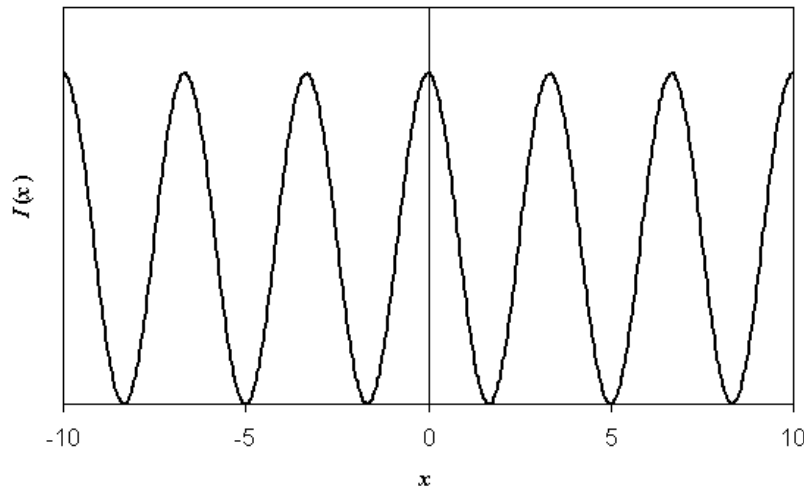
When using Solver, it is in your interests to choose units that maximize the numeric value of $\Sigma(\text{error}^2)$ to minimize rounding errors. Therefore, make sure that your intensities are proportional to $(\text{mV})^2$ rather than $(\text{V})^2$.

Theory

Interference: First let us consider the case of two coherent point sources separated by a distance d . Waves from the sources are made to interfere along a line a distance L from the sources. Assuming that $x \ll L$, the intensity as a function of the distance x is given by

$$I = 4I_0 \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta \right),$$

where I_0 is the intensity from just one of the sources by itself at the same distance, as shown in figure below:



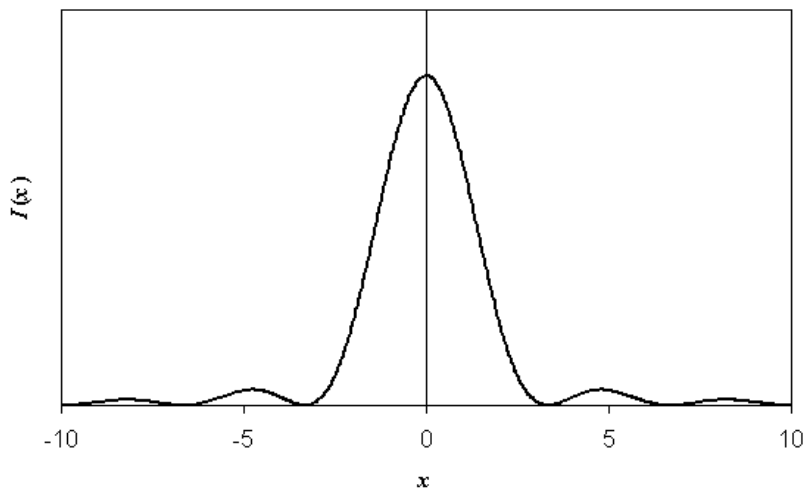
The intensity maxima occur at those angles θ that satisfy the condition

$$\frac{d}{\lambda} \sin \theta = m, \quad \text{where } m = 0, 1, 2, 3, \dots$$

Diffraction: Long rectangular slit: A diffraction pattern is produced when waves from different parts of the same source superimpose at a distance. Any source of finite width a will produce a diffraction pattern. The intensity at a distance L for diffraction from a single source is given by

$$I = I_m \left[\frac{\sin\left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta} \right]^2,$$

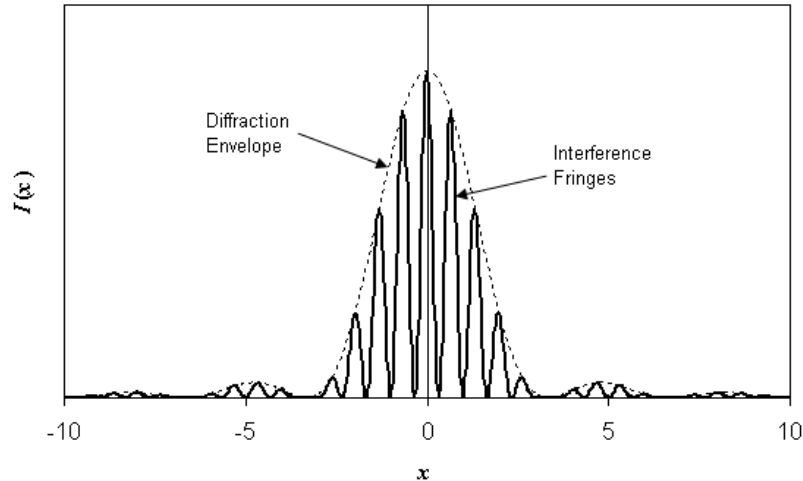
as shown in the figure below:



In this case the intensity minima occur at θ values that satisfy the equation

$$\frac{a}{\lambda} \sin \theta = m, \text{ where } m = 1, 2, 3, \dots$$

In most circumstances both these patterns will be found since in practice it is impossible to have a point source, i.e., a source that has zero diameter. The two patterns will, of course, be superimposed.



There will be a diffraction envelope and interference fringes within this envelope. Since it is simply the product of the two intensities, the intensity formula becomes:

$$I = I_m \left[\cos\left(\frac{\pi d}{\lambda} \sin \theta\right) \right]^2 \left[\frac{\sin\left(\frac{\pi a}{\lambda} \sin \theta\right)}{\frac{\pi a}{\lambda} \sin \theta} \right]^2.$$

The number of intensity maxima that fit into the central diffraction maximum is determined by the values of L , λ , a , and d . If λ is known, the size of the sources can be determined from the positions of the maxima and minima.

Also, there will typically be some background noise that must be accounted for.

Next, intensity obviously reduces with distance r^2 . That is, the farther away the receiver is, the quieter it seems. Since $r^2 = L^2 + (x - x_0)^2$, we can account for this effect by including a

dimensionless term of the form $\left(\frac{L^2}{L^2 + (x - x_0)^2} \right)$, where x_0 is the position of the point of maximum intensity.

Finally, prior experience with this lab has revealed that there is typically a shift in the center position of the diffraction envelope as compared to the interference pattern. This shift is typically a few centimeters. We can account for this by incorporating two separate “centers” for the intensity pattern: x_{0a} for the diffraction, and x_{0d} for the interference pattern.

There are several hypotheses that might account for this shift: the bench holding the receiver might not be perfectly parallel to the plane containing the two transmitters, or the strength of

the two transmitters might be slightly different, or the signals from the two transmitters might be out of phase with each other because they have slightly different resonance frequencies.

When you put together all of these portions of the analysis, the function that should describe your measured intensity pattern can be written as:

$$I = I_m \left(\frac{L^2}{L^2 + (x - x_{0d})^2} \right) \left[\cos \left(\frac{\pi d}{\lambda} \sin \theta_d \right) \right]^2 \left[\frac{\sin \left(\frac{\pi a}{\lambda} \sin \theta_a \right)}{\frac{\pi a}{\lambda} \sin \theta_a} \right]^2 + I_{\text{background}}$$

where
$$\sin \theta_d = \frac{(x - x_{0d})}{\sqrt{L^2 + (x - x_{0d})^2}}$$

and
$$\sin \theta_a = \frac{(x - x_{0a})}{\sqrt{L^2 + (x - x_{0a})^2}}.$$

You will use Excel's "Solver" feature to determine the best fit values of I_m , $I_{\text{background}}$, x_{0d} , x_{0a} , a , and d . You will also determine the uncertainties Δa and Δd .

Chapter 5: The Speed of Light

Overview:

In this experiment we will use Foucault's rotating mirror method with a laser to determine the speed of light, one of the most important physical constants in the universe!

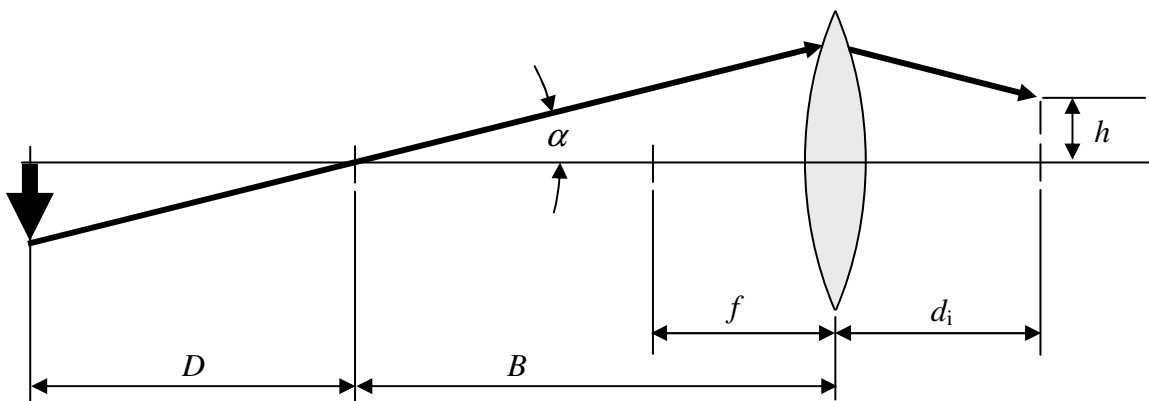
Suggested Reading Assignment:

History of the Speed of Light, see website

<http://galileoandinstein.physics.virginia.edu/lectures/spedlite.html>

Pre-lab Questions:

1. What is the exact speed of light? What is the modern uncertainty in this value?
2. What is the speed of light, expressed in miles per second?
3. How long does it take light to travel a distance of one foot?
4. We will also use lenses in this experiment. As a refresher, solve this problem: A beam of light (from a laser) is inclined at an angle of $\alpha = 0.03^\circ$, and strikes a lens having a focal length $f = 250$ mm. The beam passes the central axis of the lens while it is still $B = 500$ mm away from the lens. You can imagine that this beam originated at a distance $D + B$ from the lens, where $D = 8$ m (the fixed mirror will be located at $D + B$, and the rotating mirror at B). Compute the location d_i and the height h of the image of this beam. Also, repeat these calculations assuming that $D = \infty$. You should find that in this case, h simplifies to the multiplication of two of the givens.



Objective:

The objective of this lab is to compute the speed of light using Foucault's rotating mirror method with a laser. The primary measurement is the horizontal displacement of the beam reflected from the rotating mirror.

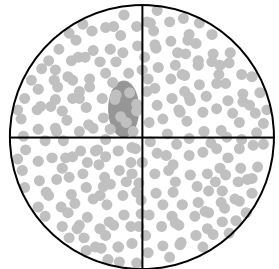
Equipment

Optical benches
HeNe Laser
252 mm convex lens with mount
48 mm convex lens with mount
Rotating mirror assembly
Large Fixed mirror
Beam splitter/telescope assembly

Experiment

1. Connect the long and short optics benches together. The millimeter markings should face you.
2. Place the rotating mirror assembly such that the front edge of the base is at 170 mm.
3. Place the laser on the short bench (i.e., past 1000 mm). Place the two L-shaped jigs on the bench, oriented with the bottom edge aimed right, as close to the mirror and the laser as possible. The back of each jig should be flush with the back fence of the optics bench.
4. Turn on the laser and adjust its horizontal angle, vertical angle, and initial position until the beam passes exactly through the hole in each jig. The vertical angle is adjusted by raising or lowering the back edge of the short optics bench.
5. Remove the jig that is near the laser and place the 48 mm lens on the bench at $x_1 = 930$ mm. Slide the lens around in the vertical plane on the magnetic mount until the beam is centered on the front jig.
6. Place the 252 mm lens on the bench near $x_2 = 622$ mm. Place an index card in front of the front jig, and adjust the lens position forwards or backwards until the spot is as small as possible. Then, remove the index card and slide the lens around in the vertical plane until the beam is centered on the front jig.
7. Remove the front jig. Place the microscope assembly on the bench so that the left edge is at 820 mm. Point the front lever down. Don't look in the telescope yet.
8. Place the large fixed mirror on a table across the room. The angle from the rotating mirror to the fixed mirror should be about 12° with respect to the optics bench. Rest an index card against the front of the mirror.
9. Rotate the rotating mirror until the beam is centered on the fixed mirror. It is very sensitive; there is a small set screw that can help the rotating mirror stay in one place. If the beam is above or below the center of the fixed mirror, shim the front or back of the rotating mirror with one or more sheets of paper. Once it is centered, remove the index card.

10. Adjust the fixed mirror until the reflected beam strikes the exact same spot on the rotating mirror as the original beam does: First, rotate the entire mirror by hand until it is very close, then use the adjustment screws on the back. This takes two people: one to adjust the mirror, and one to watch the two spots on the rotating mirror. Note that whenever you touch the fixed mirror at all, you are affecting the final position, so be sure to let go before checking the spots.
11. Do not look in the telescope. The reflected beam should be visible through it, but it is too bright and will set your brain on fire. Without looking in the telescope, adjust the front lever and the micrometer until you see a sharp red dot appear centered on the glass eyepiece of the telescope. If this doesn't work for you, you can place the two polarizers in the beam path near 950 mm and try looking through the telescope. However, be careful not to bump anything! The image should be a bright red dot, perhaps surrounded by Airy rings.
12. Place an index card behind the optics bench, near 600 mm, to block the reflected beam. Verify that the reflected spot disappears.
13. Remove the polarizers (if present) and also loosen the set screw on the rotating mirror if you used it. Your image will probably disappear.
14. Turn on the motor to start the rotating mirror at some low speed, say 200 rev/s (clockwise). The motor takes a few seconds to reach a stable speed. You can now look in the telescope. You should see a red blob, surrounded by a lot of stray interference noise. You know you're looking at the right blob if it disappears whenever you block the reflected beam (as in step 12).
15. **If the image is too noisy, try rotating the 252 mm lens mount (not the lens) a tiny amount clockwise as seen from above. Two degrees would be way too much. This step seems to always be required.**
16. Adjust the telescope lever until the spot is centered on the "vertical" line of the crosshair in the telescope.
17. Adjust the micrometer until the spot is below the crosshairs. Then, adjust the micrometer until the centroid of the spot is centered on the crosshair. Do not reverse directions as you adjust the micrometer. If you ever reverse directions, move the spot below the crosshairs and start this step over again.
18. Record both the angular speed ω of the rotating mirror and the position s of the micrometer. Be careful reading the micrometer; have your partner double check your position. It will give you four sig-figs directly, and you can estimate a fifth.
19. Vary the motor speeds from about 200 to 1000 rps in steps of about 100 rps. Record the position (see step 17) and speed (see step 18). Then, take a final measurement with the motor at 1500 rps by pressing and holding the "max speed" button. Do not let the motor spin for more than 30 seconds at this speed; it will damage the motor! Let the motor cool for one minute between attempts at maximum speed.
20. Set the rotation to counter-clockwise (CCW), and repeat the measurements. Record each CCW speed as a negative number.
21. Be sure that you have measured the distances B and D as shown below. Make one plot of s vs. ω .



Theory:

As Albert Einstein showed 100 years ago, the speed of light is the ultimate speed limit in the universe – nothing can go faster than light! The speed of light is therefore one of the most important of the fundamental physical constants, and scientists have over the years developed increasingly sophisticated techniques to measure it. Modern measurements of the speed of light have attained such high precision, in fact, that the definition of the meter is now based on the speed of light, rather than vice versa. The official definition of the meter (as of 1983) is the distance traveled by light in $1/299,792,458$ of a second (in vacuum). With this definition, it means that the speed of light is now defined to be exactly $299,792,458$ m/s, with no uncertainty. Technically, therefore, it is “impossible” to measure the speed of light because it is a defined quantity.¹ What this means is that we can in principle use a beam of light to calibrate our meter sticks. We will ignore this subtlety for the purposes of this experiment and claim to “measure” the speed of light. (Don’t you agree that calling this lab “The Speed of Light” is more interesting than calling it “A Calibration of a Meter Stick”?)

This experiment follows a method similar to Foucault’s. In summary, a laser beam is reflected from a rotating mirror. This reflected beam is then reflected again by a stationary mirror back towards the rotating mirror. Because of the travel time between the two mirrors, the rotating mirror will be at a different angular position when the beam returns. As the beam reflects off of the rotating mirror the second time, it will therefore be deflected a small amount in the direction of rotation. Additionally, a lens system will be used to focus the beam to make it easier to see it.

The beam from the laser strikes the mirror when it is at some angle θ (see Figure on page 34). Since the angles of incidence and reflection are equal, the beam is reflected at a total angle of 2θ . The beam travels to the fixed mirror and back to the rotating mirror again. However, the mirror is now rotated at an angle $\theta + \Delta\theta$. The angular velocity of the mirror is $\omega = \frac{\Delta\theta}{\Delta t}$, and the amount of time that has passed since the first reflection from the rotating mirror is $\Delta t = \frac{2D}{c}$, since the light traveled to the fixed mirror and back again. Combining these results in the expression

$$\Delta\theta = \frac{2D\omega}{c}. \quad [1]$$

The second reflection from the rotating mirror is again such that the incident and reflected angles are equal. As a result, the reflected beam returns at an angle of $2\Delta\theta$ along the optics bench.

¹ It would be like bringing in your scale from home to try to measure the mass of the platinum-iridium “standard kilogram” cylinder that is kept at the Bureau of Weights and Measures in Sevres, France... you can’t measure its mass because it is by definition equal to one kilogram. Instead you could place the standard kilogram on your scale from home in order to calibrate your scale precisely.

This returning beam is then redirected by the 252 mm lens. In the past, our examination of beams passing through lenses has been primarily about the three principal rays generated by an object. So, to make our analysis easier, we will consider our inclined beam to be only one of many rays that might have been emitted by a virtual “object” at the same distance from the lens as the fixed mirror ($D + B$ from the lens). For a ray passing through the point that is a distance B from the lens to be inclined at angle $2\Delta\theta$, the object must have an object height of $H = D \tan(2\Delta\theta)$, or, since $\Delta\theta$ is so small, simply

$$H = 2D\Delta\theta. \quad [2]$$

The object distance for this object is $D + B$, and the image distance is denoted by d_i . As usual, d_i is found from the thin lens equation $\left(\frac{1}{f} = \frac{1}{B + D} + \frac{1}{d_i}\right)$.

As we learned earlier this semester, the magnification of an object is:

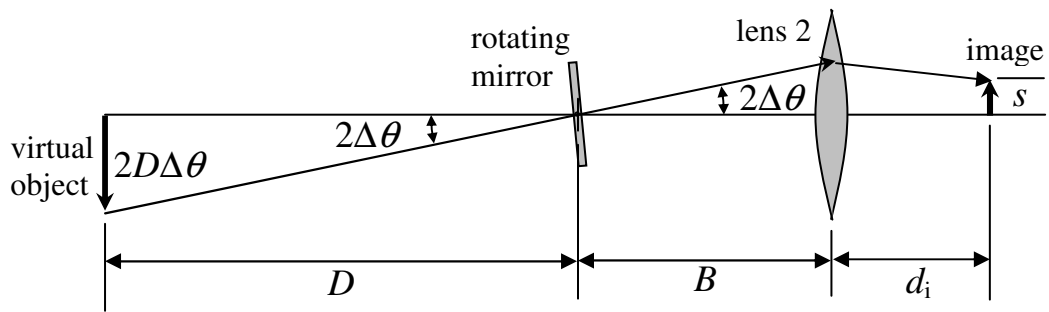
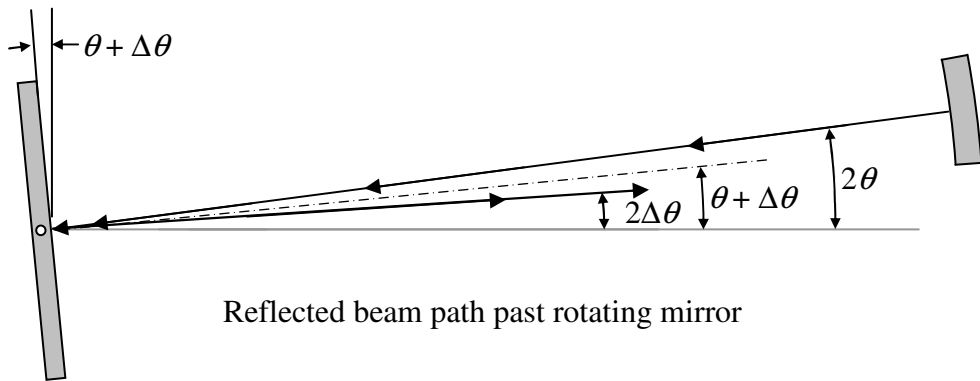
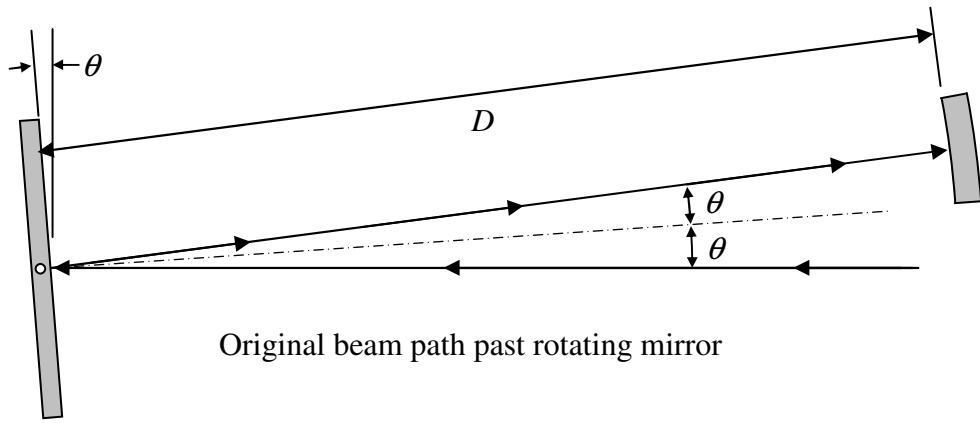
$$\left|\frac{s}{H}\right| = \left|\frac{\text{image distance}}{\text{object distance}}\right| = \frac{d_i}{D + B}, \quad [3]$$

where s is the “height” of the image. Substituting [1] and [2] into [3], and solving for s results in:

$$s = \frac{d_i}{D + B} \frac{4D^2\omega}{c}. \quad [4]$$

Because we won't bother to zero the optical axis, there will be some unknown offset for all measurements of s :

$$s = \frac{d_i}{D + B} \frac{4D^2\omega}{c} + s_0. \quad [5]$$



Chapter 6: The Bohr Atom

Overview

We will be using a grating spectrometer to study the emission spectrum of Hydrogen and compare it to that predicted by the Bohr model.

Suggested Reading Assignment

The section on the Bohr model of the hydrogen atom in your modern physics text.

E.g., Section 4-3 of Tipler and Llewellyn, 4th edition.

Pre-lab Questions

1. What is the formula for the energy levels of the Hydrogen atom as proposed by Bohr? What is the origin of the quantum number n in the equation for the energy?
2. Explain the process of absorption and emission of radiation. What is the relationship between the wavelength, the initial quantum number, and the final quantum number of the electron? Look up the value of the Rydberg constant.
3. All of the possible electron transitions that end at the same quantum number are called a “series”. What are their names of the first four series for hydrogen, and their corresponding quantum numbers? Also look up in which part of the electromagnetic spectrum these lie (i.e., infrared, visible, etc.)
4. What do H_α , H_β , H_γ , and H_δ refer to? Compute the wavelength for each, using the “accepted value” of the Rydberg constant.

Objectives

We will be using the grating spectrometer to study the emission spectrum of hydrogen and to determine the Rydberg constant predicted by the Bohr model.

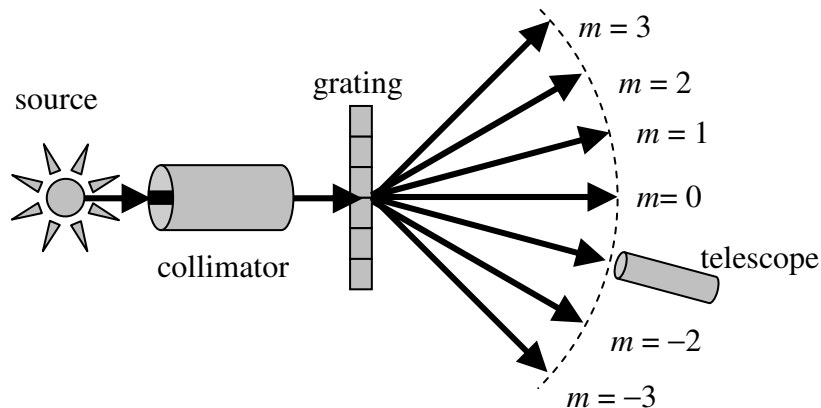
Experiment

Equipment List

- one Hydrogen spectrum lamp
- one He-Ne laser
- Grating spectrometer
- Lamps, rulers, etc.

Procedure and Analysis

The wavelengths of spectral lines can be measured using a grating spectrometer. Be sure to carefully examine the online description of this device on the course homepage before coming to lab. Light from a source (usually consisting of multiple wavelengths) passes through a collimator and then through a diffraction grating. The grating is like a double slit, but with a lot more slits. The grating separates the different wavelengths into different paths (although some light from every wavelength passes straight through, too). Furthermore, each wavelength is also split into multiple paths, called “orders”. The diagram below shows some of the possible paths for a single wavelength. The moveable telescope is viewing the ($m = -1$) order for this wavelength.



Our first goal is to determine the line spacing of the diffraction grating. Handle the grating with care (i.e., touch only the edges, and never lay it flat on the table!) Shine the beam of a Helium-Neon laser through the grating and onto a sheet of paper. Measure the spacing of the resulting diffraction pattern, and the distance from the grating to the paper. The wavelength of the He-Ne laser is 632.8 nm. Recall that for diffraction, $d \sin \theta = m \lambda$. Plot the quantity ($m \lambda$) vs $\sin \theta$ to determine d .

Now, use the spectrometer. Place the brightest, pinkest part of the hydrogen lamp a few mm in front of the slit on the collimator. At first, keep this slit wide to produce a very bright set of lines that are easy to locate. Then, before making any measurements, slowly close the slits to form a very sharp but narrow image of the slit. Note that the telescope has two focus adjustments: one, to focus the cross hairs and the other to focus the line spectrum image. Adjust the telescope until the cross-hairs are in good focus.

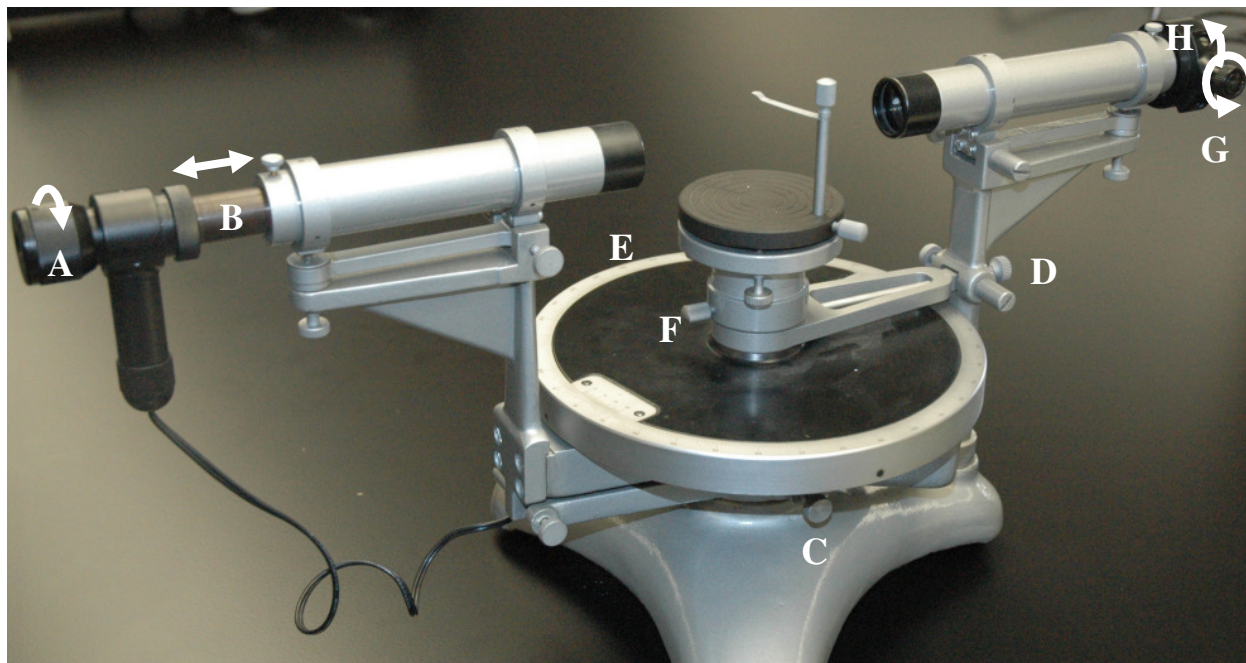
Establish the position of the 0th order (straight through) by looking at the first order on both sides ($m=\pm 1$), and finding the midpoint. Then adjust the angle indicator of the spectrometer so that this position is 0°. Please note that when m is negative, so is θ .

Move the telescope to the positions higher order images on each side and record the angular position of each order for each wavelength. The angular measurements are made using a Vernier caliper. Be sure that you are reading the angles correctly! From these angles, compute the wavelengths of the spectral lines.

For each spectral line observed, determine the Rydberg constant. (Use the information on page 32 to help determine the quantum numbers.) Use the standard deviation as the uncertainty. Is your value within experimental uncertainty?

Because of its simplicity, hydrogen is the simplest gas to analyze. As a final task, your instructor will give you a sample of a more difficult gas. Use the spectrum for this gas to identify it.

Spectrometer



Explanation of Controls

Knob A: Adjusts reticle (crosshairs); do this first.

Adjust B: Slides in/out to focus the slit.

Knob C: Locks the telescope to the base.

Knob D: Locks the black inner table to the base.

Knob E: Locks the silver outer ring (with angle markings) to the telescope.

This knob is not visible in this picture, but is similar to knob C.

Knob F: Locks the upper table (grating mount).

Knob G: Adjusts the slit width.

Knob H: Rotates the slit.

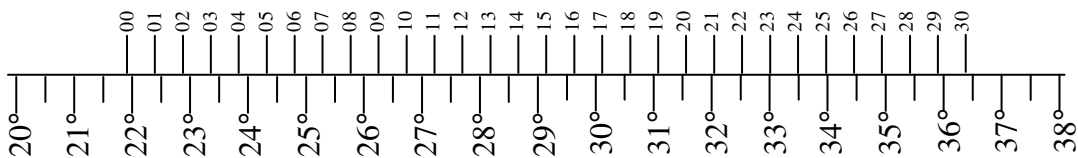
General Procedure

- Turn the inner (black) table so that the fine scale reading is somewhere on the side.
- Place the grating on the upper table, and lock it to the inner table.
- Lock the inner table to the base.
- Aim the telescope at the center of the slit (0th order... all colors).
- Turn the outer ring so that it indicates 0.00° as read by the inner table.
- Lock the telescope to the outer ring. Neither should be locked to the base.

The Vernier Scale

The scale for reading angles on the spectrometer is called a “Vernier” scale. Notice that the coarse measurement is to the nearest half a degree. Once you know the result to the next lowest half degree, use the inner scale to determine the number of **minutes** to add to your prior value. Minutes are not decimals. The number to add is the line that goes straight across.

In this example, the angle is $21^{\circ}30' + 23' = 21^{\circ}53' = 21^{\frac{53}{60}}{}^{\circ} = 21.883^{\circ}$.



Theory

The Bohr model of hydrogen (-like) atoms is based on Newtonian ideas concerning motion of a particle in a central force field, much like the motion of the planets around the sun.

However, Bohr also introduced a new *non*-Newtonian concept: quantization of angular momentum, i.e., the angular momentum of the electron moving in a circular orbit around the nucleus comes in units of the fundamental constant in quantum mechanics, \hbar .

For the hydrogen atom the allowed energies are given by

$$E_n = -\frac{13.6eV}{n^2},$$

where $n = 1, 2, 3, \dots$

When the energy of the electron goes from one value to another we refer to this transition as a “quantum jump.” When the electron makes a jump from a higher energy state (or level) to a lower one the difference in energy is emitted as light. The energy of the radiation is related to its wavelength via $E = hc / \lambda$. Thus by measuring the wavelength of the emitted radiation we can determine the difference between the energy levels. The allowed energies and transitions for the hydrogen atom are shown below.

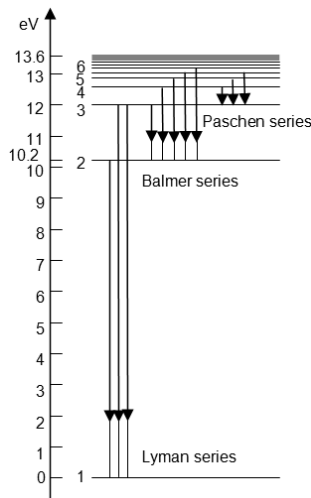
The wavelengths of the hydrogen spectrum are given by

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_{\text{lower}}^2} - \frac{1}{n_{\text{upper}}^2} \right) \quad \text{for } n_{\text{upper}} > n_{\text{lower}},$$

where

$$R_H = \left(\frac{e^2}{4\pi \epsilon_0} \right)^2 \frac{m}{4\pi \hbar^3 c} = 1.097 \times 10^7 \text{ m}^{-1}.$$

The series that lies in the visible region is known as the Balmer series, for which $n_2 = 2$ and $n_1 = 3, 4, 5, \dots$ and so on. The table below shows the Balmer wavelengths for different values of n .



| Line | n | $\lambda_{\text{computed}} (\text{\AA})$ |
|----------------|---|--|
| H_{α} | 3 | 6562.80 |
| H_{β} | 4 | 4861.33 |
| H_{γ} | 5 | 4340.48 |
| H_{δ} | 6 | 4104.75 |
| H_{ϵ} | 7 | 3970.08 |
| H_{ξ} | 8 | 3889.06 |

Chapter 7: The Michelson Interferometer

Overview

Since the index of refraction of air is so close to 1.0, it is very difficult to measure. We will use the interference patterns created by a Michelson interferometer to help us determine n_{air} to five significant digits.

Suggested Reading Assignment

The section on the Michelson Interferometer in your calculus-based physics book.

The section on the Michelson-Morley experiment in your modern physics textbook.

E.g., Section 36-8 of Halliday, Resnick, and Walker, 6th edition, and Section 1-1 of Tipler and Llewellyn, 4th edition.

Pre-lab Questions

1. Describe in your own words how the Michelson interferometer works. Draw a careful diagram.
2. The interference pattern can be made to shift if a material of thickness d is inserted in *one* of the arms of the interferometer. Derive an expression for the fringe shift ($N_{\text{material}} - N_{\text{air}}$) in terms of the thickness d , the refractive index of the material n , and the wavelength of light λ . Be prepared to solve problems based on these ideas for your pre-lab quiz.
3. Describe the Michelson-Morley experiment. This is the most famous negative experiment in physics. What was the objective of the physicists? What were the results? How did this change our view of the physical world?

Objectives

Our objective in this experiment is to measure the refractive index of air using the Michelson Interferometer.

Experiment

Equipment List

- Michelson interferometer and accessories
- Laser
- Laser Alignment bench

Procedure and Analysis

1. Align the laser and interferometer base such that the beam is approximately parallel with the top of the base. It should strike the center of mirror 1, and should be reflected directly back into the laser aperture.
2. Mount mirror 2 on the interferometer base. Position one component holder in front of the laser. Place the other component holder opposite the adjustable mirror and attach the viewing screen to its magnetic backing.
3. Position the beam-splitter at a 45° angle to the laser beam, within the crop marks, so that the beam is reflected to the fixed mirror. Adjust the angle of the beam-splitter as needed so that the reflected beam hits the fixed mirror near its center.
4. There should now be two sets of bright dots on the viewing screen; one set comes from the fixed mirror and the other comes from the movable mirror. Each set of dots should include a bright dot with two or more dots of lesser brightness (due to multiple reflections.) Adjust the angle of the beam-splitter again until the two sets of dots are as close together as possible and then tighten the thumbscrew to secure the beam-splitter.
5. Using the thumbscrews on the back of mirror 2, adjust the mirror's tilt until the two sets of dots on the viewing screen coincide.
6. Attach the 18mm lens to the magnetic backing of the component holder in front of the laser, as shown, and adjust its position until the diverging beam is centered on the beam-splitter. You should now see circular fringes on the viewing screen. If not, carefully adjust the tilt of the adjustable mirror until the fringes appear.
7. Once the circular fringe pattern appears, verify the wavelength of the laser (which we expect to be 632.8 nm). Notice the pattern of the fringes, and then slowly adjust the micrometer on the side of the apparatus. This knob moves the back mirror forwards and backwards. As the mirror moves, the fringe pattern will cycle through the bright/dark

bands until it once again appears the way it did at first. Move the mirror so that this pattern cycles ten times, and record the new mirror position. Repeat this until you have 20 measurements of microns vs. number of cycles). Plot this to compute the wavelength of the laser.

8. Place the rotating pointer/vacuum cell mount between the movable mirror and the beam-splitter. Attach the vacuum cell to its magnetic backing and push the air hose of the vacuum pump over the air outlet hole of the cell. Adjust the alignment of the fixed mirror as needed so that the center of the interference pattern is clearly visible on the viewing screen.
9. For accurate measurements, the end-plates of the vacuum cell must be perpendicular to the laser beam. Rotate the cell and observe the fringes.
10. Be sure that the air in the vacuum cell is at atmospheric pressure. This is accomplished by flipping the vacuum release toggle switch.
11. Record P_i , the initial reading on the vacuum pump gauge. Slowly pump out the air in the vacuum cell. As you do this, count N , the number of fringe transitions that occur. When you are done, record N and also P_f , the final pressure reading.

NOTE: Vacuum gauges (including ours) typically measure pressure with respect to atmospheric pressure. In this case, the absolute pressure is calculated as:

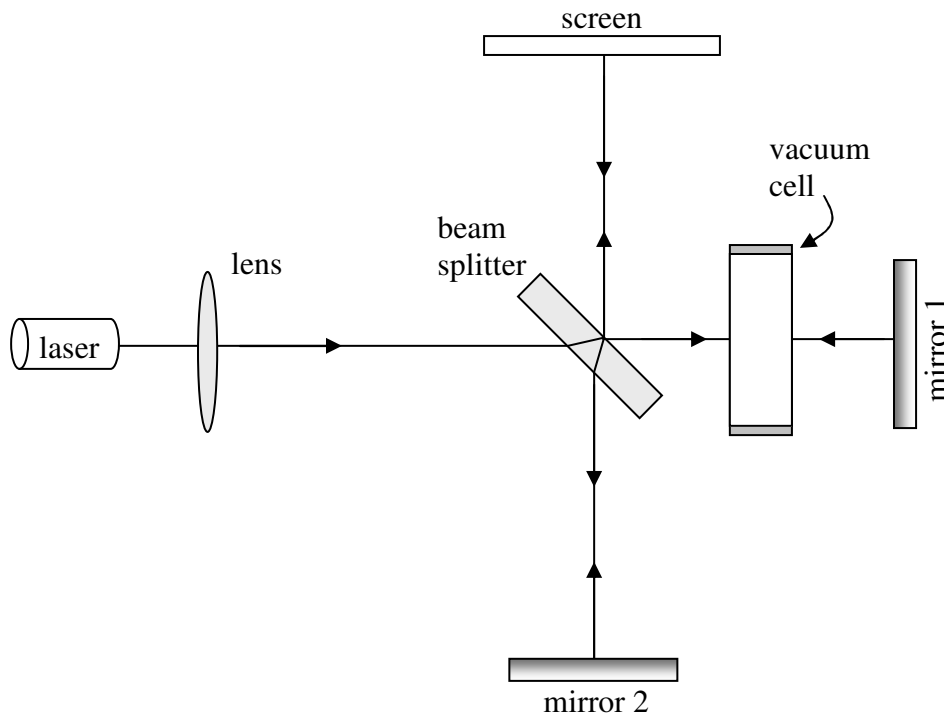
$$P_{\text{absolute}} = P_{\text{atmospheric}} - P_{\text{gauge}} .$$

Plot N as a function of $(P_i - P_f)$ and determine m (the constant of proportionality between pressure and index of refraction). Using this value of m , calculate the index of refraction for air at 1atm. Compare this with the value listed in the CRC manual for 1atm and room temperature. All results must include uncertainties.

Theory

Most of this discussion is taken from the manual that came with the PASCO Scientific Michelson Interferometer.

If two beams of light originate from the same source, there is generally some degree of correlation between the frequency and phase of the oscillations. At one point in space the light from the beams may be continually in phase. In this case, the superposition of the beams produces a maximum and a bright spot will be observed. At another point in space the light beams may be continually out of phase and a minimum, or a dark spot, will be seen. The figure below shows the Michelson interferometer apparatus.



The Michelson Interferometer

A laser beam strikes a beam-splitter, which reflects 50% of the beam and transmits the other 50%. The phases of the two resulting beams are highly correlated because they originated from the same source. One beam is transmitted toward the movable mirror 2 while the other is transmitted toward the fixed mirror 1. Both mirrors reflect the light directly back toward the beam-splitter. Half the light from mirror 1 is reflected from the beam-splitter to the viewing screen, and half is transmitted back to the laser. Similarly, half the light from mirror 2 is transmitted through the beam-splitter to the viewing screen, and half is reflected towards the laser. We will examine only the light that reaches the screen. When a lens is placed between the laser source and the beam-splitter, the light rays spread out, and an interference pattern of dark and bright rings, or fringes, is seen on the viewing screen.

The characteristics of the fringe pattern depend on the phase relationship between the two interfering beams. There are two ways to change the phase relationships. One way is to change the distance traveled by one or both beams (by moving the mirror 2, for example). Another way is to change the medium through which one or both of the beams pass. In this experiment you will use the second method to measure the index of refraction for air.

For light of a specific frequency, the wavelength λ varies according to the formula:

$$\lambda = \frac{\lambda_0}{n},$$

where λ_0 is the wavelength of the light in a vacuum, and n is the refractive index for the material in which light is propagating. In gasses at reasonably low pressures, the index of refraction varies linearly with the gas pressure. In the limit as $P \rightarrow 0$, the gas should approach vacuum, and n should approach 1.0. In equation form, this is:

$$n_i - n_f = m(P_i - P_f),$$

where n_i and n_f are the initial and final values for the index of refraction of air as the pressure is varied from P_i to P_f .

As the laser beam passes back and forth between the beam-splitter and the movable mirror, it passes through the vacuum cell twice. Outside this cell, the optical path lengths of the two interferometer beams do not change throughout the experiment. Inside the cell, however, the wavelength of the light gets longer as the pressure is reduced.

Suppose that originally the cell length, d , was 10 wavelengths long (of course, it's much longer.) As the pressure is reduced, the wavelength increases until, at some point, the cell is only $9\frac{1}{2}$ wavelengths long. Since the beam passes *twice* through the cell before reaching the screen, the light now experiences one full oscillation less within the cell, as compared to before the pressure was reduced. This has the same effect on the interference pattern as when the movable mirror is moved toward the beam-splitter by $\frac{1}{2}$ wavelength. A single fringe shift will have occurred, and the pattern will be appear the same as the starting pattern.

With the original pressure, there are $N_i = \frac{2d}{\lambda_i}$ wavelengths of light within the cell (counting

both passes of the beam). At the reduced pressure, there are $N_f = \frac{2d}{\lambda_f}$ wavelengths within

the cell. The difference between these values, $N_i - N_f$, is " N ", the number of fringes shifted as the cell was evacuated. Expressed in terms of the wavelength λ_0 ,

$$N = \frac{2d(n_i - n_f)}{\lambda_0},$$

where n_i and n_f are the initial and final values for the index of refraction of the air inside the cell.

Substituting this equation in the pressure refractive index equation, we find

$$N = \frac{2d}{\lambda_0} m(P_i - P_f).$$

So if we plot N vs. $(P_i - P_f)$ we will get a straight line from which we can determine m , given d and λ_0 . From the manufacturer, $d = 25.75 \pm 0.20$ mm.

Chapter 8: Blackbody Radiation

Overview

The objective of this experiment is to verify the Stefan-Boltzmann law (that power per unit area radiated is proportional to temperature to the fourth power), and to determine the emissivity of tungsten.

Suggested Reading Assignment

The section on “Blackbody Radiation” in your modern physics text.
E.g., Section 3-2 of Tipler and Llewellyn, 4th edition.

Pre-lab Questions

1. Explain the term “black body radiation”. Does it apply to objects that appear black only? What is “black” about a “black body”?
2. What was the “ultraviolet catastrophe”? How does quantum theory resolve this problem?
3. Look up Planck’s black body spectrum equation for the energy per unit volume, per unit frequency, in an electromagnetic field at equilibrium temperature T .
4. Starting with Planck’s radiation Law, derive the Stefan-Boltzmann law. Symbolically, what is σ in terms of h , c , and k ?
5. What is Wien’s displacement law?
6. Starting with Planck’s radiation Law, derive Wien’s displacement law for the wavelength at which the black body energy density is a maximum. You will need to numerically solve a transcendental equation (perhaps using Excel’s “solver”). Be sure that your numerical result is accurate to at least three significant figures.

Experiment

Equipment List

- #47 Light Bulb
- Ammeter
- Digital Multimeter
- Pyrometer
- 20 Volt Power Supply

Procedure

Part 1: Look up the resistivity of tungsten as a function of temperature in the CRC handbook (see Dr. Pogo's web page...). For convenience, we'll do some preliminary algebraic manipulations. Think of it as a function of temperature versus resistivity. Define a relative

resistivity $\rho' = \frac{\rho}{\rho_{300\text{K}}}$, where ρ is the resistivity at temperature T and $\rho_{300\text{K}}$ is the resistivity

at 300 K (room temperature). Recall that $R = \frac{\rho L}{A}$, where L is the length of the object and

A is its cross sectional area. So, for any given wire, if A and L are constant, then

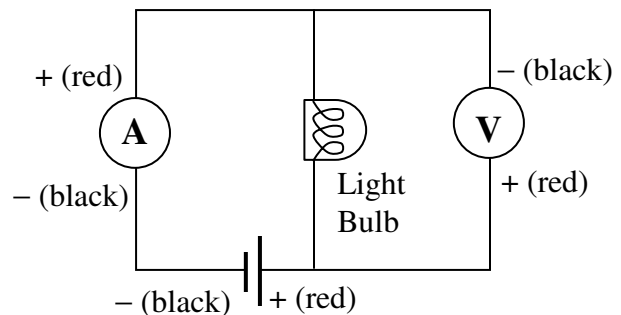
$\frac{\rho}{\rho_{300\text{K}}} = \frac{R}{R_{300\text{K}}}$, so $\rho' = R' = \frac{R}{R_{300\text{K}}}$. Make a plot of T versus R' and fit these data with an

appropriate function (a third order polynomial should be fine). What is the typical error in the predicted temperature using your fitted result?

Part 2: Use the ohmmeter to record the resistance of the unlit light bulb at room temperature. Use the most sensitive setting on the ohmmeter. The resistance in the bulb will be small, and you'll have to make sure your answer does not include the resistance of the wires themselves. Your final answer will be very sensitive to this measurement, be sure you perform it with care! The trick is to make sure you have very, very tight connections. Loose connections make the resistance seem higher than it really is.

Next, connect the apparatus as shown in the figure below. Make sure that you use the proper polarity on the ammeter... **failure to do so will cause permanent damage to these sensitive (and expensive) pieces of equipment!**

Record the voltage across the light bulb as a function of the current through the light bulb. Take about 30 data points, from 0 to 15 volts in increments of about 0.5 volts or so. Note that the tungsten filament may begin to vaporize at the high voltages; therefore it is important that you try to move as quickly as possible at the high voltages.



Analysis

Enter your voltage and current data into Excel. For each data point, determine the resistance of the light bulb (note that the resistance should increase as the filament gets hotter!). Then, from the current and voltage across the bulb, make another column on excel in which you determine the electrical power dissipated by the light bulb (you should remember how to calculate this from Analyt II). For each resistance value, determine the temperature of the light bulb using the relationship you found in part I.

According to the Stefan-Boltzmann Law, P/T^4 should be a constant for an ideal blackbody. Make a plot of P/T^4 versus temperature. Is there a region where this ratio stays relatively constant? You may have to discard some data on the left side that doesn't. If it is not flat, then you need to restart from the beginning. Do not proceed until this looks flat!

Plot P vs. T^4 for the data suggested by the previous plot. According to the Stefan Boltzmann Law, this should look like a straight line... does it? Fit these data with a straight line.

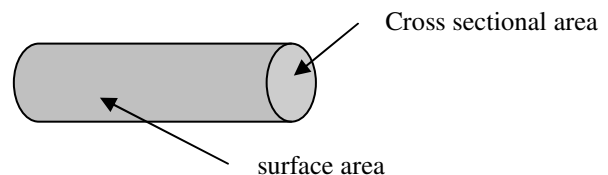
Next, verify the Stefan-Boltzmann Law by determining how the power emitted by the blackbody depends on the temperature. Suppose that the power depends on temperature as $P = aT^n$. Our job then is to find the value of n (theoretically, it should be 4.) We take the natural log of both sides to obtain

$$\ln(P) = \ln(a) + n \ln(T)$$

Evidently, plotting $\ln(P)$ vs. $\ln(T)$ will allow us to calculate the value of n (and a coarse value for a). Fit an appropriate region of the data to determine n . How does your value agree with the accepted value (within uncertainty)?

Finally, let's determine the emissivity of tungsten, using the known Stefan-Boltzmann constant, σ . First determine the ratio $\frac{L}{A}$ using the definition of resistivity: $R_{300\text{K}} = \frac{\rho_{300\text{K}} L}{A}$.

You can combine these results to find the cross sectional area of the filament. This will allow you to estimate the radius of the filament, from which you can find the total surface area of the filament (make sure you distinguish between the *cross sectional* area in the resistivity formula and the total surface area in the Stefan-Boltzmann Law). Now that you know the surface area of the filament, you use the slope of the P vs. T^4 graph to obtain the emissivity of tungsten. Does your result agree with the accepted result (0.43)?



Theory

An ideal black body radiates energy according to the Planck Radiation Law, which states that the power emitted per unit surface area in the frequency range between (f) and ($f + df$) is

$$J(f, T) df = \frac{c}{4} u(f, T) df = \frac{2\pi hf^3}{c^2} \frac{1}{e^{hf/kT} - 1} df$$

where $u(f, T)$ is the spectral energy density, c is the speed of light, k is Boltzmann's constant, h is Planck's constant, and T is the temperature of the object. The total power per unit area radiated at all frequencies can be found by integrating this result over all frequencies,

$$\frac{P}{A} = \int_0^{\infty} J(f, T) df = \frac{2\pi h}{c^2} \int_0^{\infty} \frac{f^3}{e^{hf/kT} - 1} df = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \int_0^{\infty} \frac{u^3}{e^u - 1} du = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15}$$

The above equation can be rewritten as

$$\frac{P}{A} = \sigma T^4$$

where the constant σ is given by

$$\sigma = \frac{2\pi k^4 \pi^4}{15h^3 c^2} = 5.67 \times 10^{-8} \text{ W / m}^2 \text{ K}^4$$

This result is known as the Stefan-Boltzmann Law, and σ is known as the Stefan-Boltzmann constant.

For a “grey” body, the Stefan-Boltzmann Law instead becomes:

$$\frac{P}{A} = \epsilon \sigma T^4,$$

where ϵ is the emissivity of the material.

Chapter 9: Photoelectric Effect

Overview

Suggested Reading Assignment

The section on “The Photoelectric Effect” in your modern physics text.
E.g., Section 3-3 of Tipler and Llewellyn, 4th edition.

Pre-lab Questions

1. Explain the photoelectric effect and Einstein’s theory of the photoelectric effect.
2. Why does the existence of a cutoff frequency ν in the photoelectric effect favor a particle theory for light rather than a wave theory?
3. What effect, if any, would you expect the temperature of a material to have on the ease with which electrons can be ejected from it in photoelectric effect?
4. What frequency of light is needed to just barely liberate electrons from an aluminum surface?
5. What wavelength of light is needed to liberate electrons from a nickel surface so that they have kinetic energy 2.0 eV? Note: for this lab, and for all nuclear, atomic, or quantum calculations you might make in future coursework, energy should be given in eV rather than Joules. Do not convert to Joules for this question, or on your pre-lab quiz, or in your Journal for this lab.
6. The filters we’re using specify an average value, along with a FWHM value. What does this stand for? What does it mean? How does it differ from a standard deviation?

Objectives

The objective of this experiment is to use the photoelectric effect to determine Planck's constant, and also to determine the work function of the metal in our apparatus.

Experiment

Equipment List

- Photoelectric devices
- Filters in a folder with the specs
- High intensity lamps
- Digital multimeters

Procedure and Analysis

Return each filter to its corresponding folder before getting the next one. Record both the average wavelength and the FWHM value for each filter, so you can determine a “minimum” wavelength. This is the value that we will use in the analysis later on.

Set up the photoelectric device so that the aperture faces the lamp. The devices are sensitive to excessive light, so be careful not to shine the lamp directly on the device without a filter or a piece of cardboard protecting it. Place one of the filters in front of the aperture by sliding it between the two “clips”. Handle the filters with care. Among other things, do not touch their faces; handle them by the edges only. With the filter in place, position the lamp close to the aperture. Connect the digital multimeter to the photoelectric device to record voltage.

For *every* filter: First, cover the aperture with a piece of cardboard, and then turn the “voltage adjustment” knob to its clockwise limit. Then, adjust the “zero adjustment” knob until the current meter reads zero. Then, turn the “voltage adjustment” knob back to its counterclockwise limit and replace the cardboard with a filter (rounded edge down). Turn on the lamp. To ensure consistency in our “definition of zero” (see below), adjust the lamp position until the current is some preselected value (perhaps 10nA). It doesn't matter much what the value is, as long as it is the same for every filter. The lamp should be far enough away so that it doesn't heat the apparatus.

Now adjust the voltage to just make the current 2 nA, 1 nA, $\frac{1}{2}$ nA, and 0 nA (as best as you can tell). Repeat each measurement three times, recording the corresponding voltages. The stopping potential is the voltage when the current *just* reaches zero, which is all we really want. However, this value can be hard to determine, because the stopping potential changes relatively rapidly as the current approaches zero. Furthermore, the lamp will slowly heat the metal, changing the work function for the metal. So the longer you have the source light on, the worse your data will become.

When you've used all the filters, plot a graph of the stopping voltage versus the inverse of the wavelength. Use "linest" in Excel to get the slope, the intercept, and the uncertainties.

From your data determine Planck's constant and the work function of the material. Look in the CRC manual (or any online source) for typical values of work functions for different materials. Does yours agree with any of them? Explain.

Theory

When light is made to shine on the surface of a metal (or alloy) the electrons bound in the lattice of the metal acquire enough energy from the striking photons to escape from the lattice. The emitted electrons can be directed towards a positively charged anode and be detected as a current when the anode and cathode are connected through an external circuit. The electron emission is strongly dependent on the frequency ν of the incident light. For each metal there is a critical frequency ν_0 such that light of frequency less than this frequency can never liberate electrons, while light of frequency greater than this value has enough energy to do this. As the frequency of the incident radiation is increased the energy of the emitted electrons increases linearly. The intensity of the light affects only the number of emitted electrons, not their energy.

The Einstein photoelectric equation reads:

$$E_{\max} = h\nu - \phi$$

The above equation states that the maximum possible kinetic energy (E_{\max}) of the emitted electron is the energy of the incoming photon minus the amount the electron needs to escape from the metal. The quantity ϕ is known as the work function of the metal (which, of course, varies from metal to metal.) If the incident light has several frequencies, the highest frequency determines E_{\max} .

The critical frequency is that frequency for which the energy of the incoming photon just balances the energy required for the electron to escape, i.e.,

$$\nu_0 = \frac{\phi}{h}$$

In order to measure the maximum energy of the electrons, an external potential difference is applied between the anode and the cathode which will retard the flow of electrons from cathode to anode. When this retarding potential is enough to stop the most energetic electrons no current flows in the circuit. Thus we have,

$$eV_0 = h\nu - \phi,$$

where V_0 is the stopping potential. Rewriting this equation, we have

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$

where λ is the minimum wavelength of the incident light.

Chapter 10: Chaos – Computer Simulation

Overview

To study the chaotic behavior of a numerically simulated non-linear system.

Suggested Reading Assignment

http://en.wikipedia.org/wiki/Chaos_theory

http://en.wikipedia.org/wiki/Logistic_equation

<http://www.around.com/chaos.html>

Pre-lab Questions

None!

Theory

Some physical systems that are governed by non-linear equations exhibit what is known as chaos. When we think of the word “order”, we associate with it systems that are periodic, predictable, well behaved, etc. A *predictable* system is one in which, if the initial conditions are known, the time evolution can be determined with certainty. For example, an object (say, a ball) undergoing projectile motion is such a system. Given the initial position and velocity, its trajectory can be predicted as a function of time.

H. Bruce Stewart, an applied mathematician at Brookhaven National Laboratory on Long Island defines chaos as: “Apparently random recurrent behavior in a simple deterministic (clockwork-like) system.” In other words, systems that one would expect to behave in a predictable manner appear quite random at first glance---but then under closer examination, turn out to be orderly in a most unusual way. The phenomenon of chaos can be found in different situations ranging from the weather, the shapes of clouds, the shapes of snowflakes, commodity prices, electrical circuits, heartbeat patterns animal populations and so on.

Experiment

Equipment List

This experiment is a computer simulation.

Procedure and Analysis

The system we will consider is a model for determining animal population. There are many factors that affect growth (or decline) of animal population. Firstly, the population of a certain species in a given year depends on the previous year’s population. So the first simple step towards building a model is the equation:

$$x_{\text{next}} = F(x_{\text{present}}),$$

where x is the population in a given year, and this year’s population depends on its value last year. A reasonable form for this function would be one in which the populations in successive years are assumed to be simple ratios. In other words, they could be related via a linear functional dependence, such as,

$$x_{n+1} = \mu x_n,$$

where x_{n+1} represents next year’s population and x_n represents this year’s. However, if this were correct, the population would grow indefinitely! A more realistic model would be one

that would account for attrition due to starvation, predators, etc. in the formula as a way to stabilize the population. The equation

$$x_{n+1} = \mu x_n (1 - x_n)$$

does exactly this; the factor $(1 - x_n)$ decreases as x_n increases. Note that the population is normalized such that it only takes on values from 0 to 1. This equation is called the “logistic equation”.

In this computer experiment, we will model this specific equation and study the dependence of the value of x_n on the parameters μ , n , and the relationship between successive years’ populations. Assume that μ is between 0 and 4.

- A. Make a plot of the current year’s population versus the previous year’s population, i.e., x_{n+1} versus x_n , for various values of μ . Choose $\mu = 0.6, 1.5, 2.9, 3.25, 3.54, 3.833,$ and 3.99 . Put all the plots on one set of axes. What is the expected shape of these graphs, and why?
- B. Create seven separate plots (all arranged on one page, please) of predicted population (i.e., x_n) vs. time for 200 years starting with a random initial population for different values of the parameter $\mu = 0.6, 1.5, 2.9, 3.25, 3.54, 3.833,$ and 3.99 . How is the long term population affected by the choice of the initial population? How predictable is the long future population for each of these cases? In each case, how many possibilities exist for the long term population?
- C. Since we see that at large times, the final population is not significantly related to the original population, let’s examine this idea using a lot more values of μ than the mere 6 values used in part B. Create a plot of x_n versus μ for $0 < \mu < 4$, for large times. Your instructor may give you more specific instructions. Be warned! This exercise will strain the limits of Excel, and Excel may experience significant time delays as a result. Have some patience! This plot should show ALL possible final populations, for all values of μ and x_0 . The plots of part B can each be thought of as vertical slices of this new plot, for individual values of μ .
- D. Zoom in on a section having a bifurcation branch by adjusting the axis scales. Once you have identified a nice branch, create an entirely new plot for this region, using an even finer distribution of μ values (again, your instructor may give you more detailed instructions). Compare this new plot to the plot of part C. The fact that such repetitions can be found at smaller and smaller scales indicates that this plot is a *fractal*.